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GPS Signal Detection Using Hypothesis Testing Analysis

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Abstract

GPS signal detection using hypothesis testing analysis are given by using the generalized likelihood ratio test (GLRT) approach, applying the model of intermediate frequency (IF) GPS signal of one satellite in white Gaussian noise. The test statistic follows central or noncentral F distribution and is nearly identical to central or noncentral chi-squared distribution because the processing samples are large enough to be considered as infinite in GPS acquisition algorithms. The probability of false alarm, the probability of detection and the threshold are affected largely when the hypothesis testing refers to the full PRN code phase and Doppler frequency search space cells instead of to each individual cell. The performance of the test statistic is also given with combining the noncoherent integration. Given the probability of false alarm to achieve a desired probability of detection, examples are illustrated to determine the relations among the threshold, the coherent integration time, the number of noncoherent integration and signal to noise ratio.

Keywords: GPS, Hypothesis Testing, GLRT, Signal Detection.

1. Introduction

Detection (Kay, 1993-1) and estimation (Kay, 1993-2) are two aspects for signal processing. The received Global Positioning System (GPS) signal is buried in noise. We are interested in determining the presence or absence of satellite signals (detection aspect) and unknown parameter estimation (estimation aspect). The GPS signal detection can be based on a hypothesis testing that could be summarized as hypothesis $\mathcal{H}_1$, some satellite signal is present and hypothesis $\mathcal{H}_0$, it is not. $\mathcal{H}_0$ is referred to as the null hypothesis and $\mathcal{H}_1$ as the alternative hypothesis. This problem is known as a binary hypothesis test since we must choose between two hypotheses (Kay, 1993-1). In Bromberg and Progri (2004), Bayesian estimation techniques are applied to the problem of time and frequency offset estimation for GPS receivers. The estimation technique employs Markov Chain Monte Carlo (MCMC) to estimate unknown system parameters. In Progri et al. (2003), it proposes a maximum likelihood GPS receiver for processing the received GPS signals of the L1 and L2 frequencies. The maximum likelihood GPS receiver performs a simultaneous, two-dimensional search of both the PRN code phase and Doppler frequency. In Winternitz et al. (2004), O'Driscoll (2007), and Shanmugam (2008), the GLRT approach is applied, but not that much in detail. In this paper, we also resort to GLRT approach to detect the GPS signal. Because the variance of WGN is unknown but has been taken into consideration under both hypotheses $\mathcal{H}_1$ and $\mathcal{H}_0$, the resultant hypothesis test leads to doubly composite hypothesis testing problem (Kay, 1993-1).

In Dierendonck (1996), Ziedan (2006), Psiaki (2001) and Hegarty et al. (2003), they use the conclusion that the GLRT test statistic of GPS signal will follow central and non-central chi-squared distribution under hypotheses $\mathcal{H}_1$ and $\mathcal{H}_0$, respectively. In this paper, based on the theorems (Kay, 1993-1) shown in Appendix, it has proved that in fact the test statistic follows central or noncentral F distribution. It has also shown that the test statistic is nearly identical to central or noncentral chi-squared distribution only because the processing samples are large enough to be considered as infinite in GPS signal detection algorithms.

In Dierendonck (1996), the hypotheses $\mathcal{H}_1$ and $\mathcal{H}_0$ refer to each individual cell, and not to the full search space. And thus has the conclusion that increasing noncoherent integration number does not change the threshold. As a consequence, there is a high statistical risk that the noise will be, in some cells, higher than the calculated threshold. In this paper, it has proved that the probability of false alarm, the probability of detection and the threshold are all affected largely when the hypothesis testing refers to the whole PRN code phase and Doppler frequency search space cells instead of to each individual cell.

In this paper, as the test statistic considering all search space cells, the performance of the hypothesis testing is also given with combining the noncoherent integration to increase the processing gain.

For different acquisition methods, the expression of the probability of false alarm, the probability of detection and the threshold will be different (Ziedan, 2006; Psiaki, 2001; Borio et al., 2006). In this paper, we have derived the basic expression which can be altered and then applied to different acquisition methods.

The rest of the paper is organized as follows. First, hypothesis testing analysis of GPS is introduced. Second, performance analysis is given. Third, GPS signal detection with noncoherent integration is analysed. Forth, given the probability of false alarm to achieve a desired probability of detection, examples are illustrated to determine the relations among the threshold, the coherent integration, the number of noncoherent integration and signal to noise ratio. Finally, conclusions are made.

2. Hypothesis Testing Analysis

Considering the detection of received sampled GPS intermediate frequency (IF) signal of one satellite in WGN and assuming the IF signal has a sampling frequency of f_s, the detection problem becomes

$$\begin{aligned} &\mathcal{H}_0 : x[n] = w[n] && \sigma^2 > 0, n = 0,1,\cdots,N-1 \\ &\mathcal{H}_1 : x[n] = Ad(n,n_\tau,f_d)C(n,n_\tau,f_d) && \sigma^2 > 0, n = 0,1,\cdots,N-1 \\ &\quad \cos(2\pi(f_{IF}+f_d)nT_s+\phi)+w[n] && \end{aligned} \tag{1}$$

where A is the signal amplitude; $T_s = 1/f_s$ is the sampling period; $S/N_0 = A^2/(2\sigma^2 T_s)$ and the relation between S/N_0 and C/N_0 is $S/N_0 = 10^{(C/N_0)/10}$, C/N_0 is the carrier to noise ratio in dB-Hz units; d is the navigation data, with a data bit rate of 50Hz; C is the received PRN code, which has a length of 1023 chips and a chipping rate of 1.023MHz; ϕ is the initial phase of the carrier signal; f_{IF} is the IF carrier frequency; f_d is the Doppler frequency shift; n is the index of samples, and N is the number of the samples; n_τ is the PRN code delay; $w[n]$ is WGN with variance σ^2 and zero mean. Parameters $\{A,\phi,f_d,n_\tau,\sigma^2\}$ are unknown. In either case, the resultant hypothesis testing has unknown parameters under both $\mathcal{H}_0$ and $\mathcal{H}_1$ due to the noise parameters. It is termed the doubly composite hypothesis testing problem (Kay, 1993-1).

The sequence of PRN code is known for an appointed satellite, while the sequence of the navigation data is unknown. One navigation data lasts for 20ms, during which time it includes 20 PRN code periods. So there are 20 possible bit edges with each one is aligned with the start of a 1ms PRN code period. Taking Doppler frequency shift on the length of PRN code into consideration, the sampling time of the received PRN code and the data is expressed as follows

$$t_{n,n_\tau,f_d} = (t_n - t_{n_\tau})(1+f_d/f_{L1}) = T_s(n-n_\tau)(1+f_d/f_{L1}) \tag{2}$$

where t_n is the nth sample time; t_{n_τ} is the time of the code delay corresponding to the n_τth sample; f_{L1} is the L1 carrier frequency.

Suppose that a two dimensional search will be done over the full code phase uncertainty of 1023 chips and a Doppler frequency shift uncertainty range from f_{dmin} to f_{dmax}. The number of code phase search over 1023 chips equals to the number of samples over 1ms, $N_\tau = f_s T_{1ms}$, where $T_{1ms} = 1\text{ms}$. For PIT T_I, the number of Doppler frequency cells is $N_{f_d} = (f_{dmax} - f_{dmin})T_I$. So the total number of cells to be searched by considering all possible code phase and Doppler frequency shift combinations is $N_{search} = N_\tau N_{f_d}$.

Suppose the data bits and bit edge in T_I are known as a priori, $d(n,n_\tau,f_d)$ is then can be neglected in below. So hypothesis $\mathcal{H}_1$ becomes

$$\mathcal{H}_1 : x[n] = AC(n,n_\tau,f_d)\cos(2\pi(f_{IF}+f_d)nT_s+\phi)+w[n] \tag{3}$$

Because A and ϕ are unknown, we must assume that $A>0$. Otherwise, two different set of A and ϕ, i.e, $A=1,\phi=0$ and $A=-1,\phi=\pi$, will yield the same signal, and thus the parameters will not be identifiable (Kay, 1993-1).

We can rewrite $x[n]$ as

$$\begin{aligned} x[n] = &\alpha_1 C(n,n_\tau,f_d)\cos(2\pi(f_{IF}+f_d)nT_s)+ \\ &\alpha_2 C(n,n_\tau,f_d)\sin(2\pi(f_{IF}+f_d)nT_s)+w[n] \end{aligned} \tag{4}$$

where $\alpha_1 = A\cos(\phi)$, $\alpha_2 = -A\sin(\phi)$. Clearly, $A=0$ if and only if $\alpha_1 = \alpha_2 = 0$ since $A = \sqrt{\alpha_1^2+\alpha_2^2}$. Thus we have the detection problem equivalent to

$$\begin{aligned} \mathcal{H}_0 &: x[n] = \alpha_1 C(n, n_\tau, f_d)\cos(2\pi(f_{IF} + f_d)nT_s) + \\ &\quad \alpha_2 C(n, n_\tau, f_d)\sin(2\pi(f_{IF} + f_d)nT_s) + w[n] \\ &\quad \alpha_1 = \alpha_2 = 0, \sigma^2 > 0, n = 0,1,\cdots,N-1 \\ \mathcal{H}_1 &: x[n] = \alpha_1 C(n, n_\tau, f_d)\cos(2\pi(f_{IF} + f_d)nT_s) + \\ &\quad \alpha_2 C(n, n_\tau, f_d)\sin(2\pi(f_{IF} + f_d)nT_s) + w[n] \\ &\quad \alpha_1^2 + \alpha_2^2 \neq 0, \sigma^2 > 0, n = 0,1,\cdots,N-1 \end{aligned} \quad (5)$$

3. Performance Analysis

3.1 If code phase and Doppler shift are known

Firstly, we only take consideration of unknown parameters A, ϕ, σ^2 and assume that n_τ, f_d are known. In terms of the linear model we have

$$\mathbf{x} = \mathbf{H}\boldsymbol{\theta} + \mathbf{w} \quad (6)$$

Where

$$\mathbf{H} = \begin{bmatrix} C(0, n_\tau, f_d) & 0 \\ C(1, n_\tau, f_d)\cos(2\pi(f_{IF} + f_d)T_s) & C(1, n_\tau, f_d)\sin(2\pi(f_{IF} + f_d)T_s) \\ \vdots & \vdots \\ C(N-1, n_\tau, f_d)\cos(2\pi(f_{IF} + f_d)(N-1)T_s) & C(N-1, n_\tau, f_d)\sin(2\pi(f_{IF} + f_d)(N-1)T_s) \end{bmatrix} \quad (7)$$

$$\boldsymbol{\theta} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \quad (8)$$

Then the GLRT for the hypothesis testing problem becomes

$$\begin{aligned} \mathcal{H}_0 &: \boldsymbol{\theta} = 0, \sigma^2 > 0 \\ \mathcal{H}_1 &: \boldsymbol{\theta} \neq 0, \sigma^2 > 0 \end{aligned} \quad (9)$$

According to Theorem 9.1 (Kay, 1993-1) in Appendix, we have $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\mathbf{b} = 0$ in this problem. Then we decide $\mathcal{H}_1$ if

$$T_F(\mathbf{x}) = \frac{(N-2)}{2} \frac{\left(\mathbf{A}\hat{\boldsymbol{\theta}}_1\right)^T \left(\mathbf{A}\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{A}^T\right)^{-1} \mathbf{A}\hat{\boldsymbol{\theta}}_1}{\mathbf{x}^T\left(\mathbf{I} - \mathbf{H}\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\right)\mathbf{x}} > \gamma_F \quad (10)$$

where

$$\hat{\boldsymbol{\theta}}_1 = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{x} \quad (11)$$

is the MLE of $\boldsymbol{\theta}$ under $\mathcal{H}_1$. γ_F is the threshold of (10).

Using

$$C^2(n, n_\tau, f_d) = 1 \quad (12)$$

$$\sum_{n=0}^{N-1}\sin\left(2\pi\left(f_{IF} + f_d\right)nT_s\right)\cos\left(2\pi\left(f_{IF} + f_d\right)nT_s\right) \approx 0 \quad (13)$$

$$\begin{aligned} \sum_{n=0}^{N-1}\cos^2\left(2\pi\left(f_{IF} + f_d\right)nT_s\right) &\approx \frac{N}{2} \\ \sum_{n=0}^{N-1}\sin^2\left(2\pi\left(f_{IF} + f_d\right)nT_s\right) &\approx \frac{N}{2} \end{aligned} \quad (14)$$

$\mathbf{H}^T\mathbf{H}$ becomes

$$\mathbf{H}^T\mathbf{H} \approx \frac{N}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{N}{2}\mathbf{I}_{2\times 2} \quad (15)$$

Then

$$\hat{\boldsymbol{\theta}}_1 = \left(\frac{N}{2}\mathbf{I}_{2\times 2}\right)^{-1}\mathbf{H}^T\mathbf{x} = \frac{2}{N}\begin{bmatrix} \sum_{n=0}^{N-1} x[n]C(n, n_\tau, f_d)\cos\left(2\pi\left(f_{IF} + f_d\right)nT_s\right) \\ \sum_{n=0}^{N-1} x[n]C(n, n_\tau, f_d)\sin\left(2\pi\left(f_{IF} + f_d\right)nT_s\right) \end{bmatrix} \quad (16)$$

The exact detection performance is given by

$$P_{FA} = Q_{F_{2,N-2}}(\gamma_F) \quad (17)$$

$$P_D = Q_{F'_{2,N-2}(\lambda_F)}(\gamma_F) \quad (18)$$

where $F_{2,N-2}$ denotes an F distribution with 2 numerator degrees of freedom and $N-2$ denominator degrees of freedom (Kay, 1993-2). And $F'_{2,N-2}(\lambda_F)$ denotes a noncentral F distribution with 2 numerator degrees of freedom, $N-2$ denominator degrees of freedom and noncentrality parameter λ_F which is given by

$$\lambda_F = \frac{\boldsymbol{\theta}_1^T\mathbf{H}^T\mathbf{H}\boldsymbol{\theta}_1}{\sigma^2} = \frac{NA^2}{2\sigma^2} \quad (19)$$

where $\boldsymbol{\theta}_1$ is the true value of $\boldsymbol{\theta}$ under $\mathcal{H}_1$. Because $S/N_0 = A^2/\left(2\sigma^2 T_s\right)$ and $NT_s = T_I$, we have $\lambda_F = (S/N_0)T_I$.

The discussion of the F distribution can be resort to (Kay, 1993-1). When $N \to \infty$, we have $F_{2,N-2} \to \chi_2^2/2$ and $F'_{2,N-2}(\lambda) \to \chi_2'^2(\lambda)/2$. Then the test statistic is nearly identical to that of Theorem 7.1 (Kay, 1993-1) in Appendix. The principle difference (apart from the scale factor 2) is that the denominator σ^2 has been replaced by its unbiased estimator

$$\hat{\sigma}_1^2 = \frac{1}{N-2}\mathbf{x}^T\left[\mathbf{I} - \mathbf{H}\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\right]\mathbf{x} \quad (20)$$

Then (10) becomes

$$T_F(x)=\frac{\hat{\boldsymbol{\theta}}_1^{\,T}\mathbf{H}^T\mathbf{H}\hat{\boldsymbol{\theta}}_1}{2\hat{\sigma}_1^2}$$

$$=\frac{1}{N\hat{\sigma}_1^2}\left\{\begin{array}{l}\left[\sum_{n=0}^{N-1}x[n]C(n,n_\tau,f_d)\cos(2\pi(f_{IF}+f_d)nT_s)\right]^2+\\ \left[\sum_{n=0}^{N-1}x[n]C(n,n_\tau,f_d)\sin(2\pi(f_{IF}+f_d)nT_s)\right]^2\end{array}\right\} \quad (21)$$

$$=\frac{1}{N\hat{\sigma}_1^2}\left|\sum_{n=0}^{N-1}x[n]C(n,n_\tau,f_d)\exp(-2\pi(f_{IF}+f_d)nT_s)\right|^2>\gamma_F$$

Now we decide whether the sample number N can be considered as infinite or not in GPS signal acquisition problem. The PDF of central F distribution is denoted by F_{v_1,v_2}, and noncentral F distribution $F'_{v_1,v_2}(\lambda_F)$. Suppose $v_1=2$, $C/N_0=30\text{dB}-\text{Hz}$ and $T_I=1\text{ms}$, then $\lambda_F=(S/N_0)T_I=1$. We choose $v_2=100$, 1000, and ∞ separately and then draw the PDF for both central and noncentral F distribution, which are shown in Fig. 1 (a) and (b). We can see that all the three lines in each of the two figures are almost identical to each other. In GPS problem, the typical sample rate is equal or larger than f_s=2.048MHz, and with $T_I=1\text{ms}$ for normal signals or larger for weak signals, then $v_2=N-2=f_sT_I-2$ are larger than 2000. So we can say that the test statistic is identical to that when σ^2 is known and then Theorem 7.1 (Kay, 1993-1) is applied in GPS signal acquisition.
Applying Theorem 7.1, the exact detection performance is then given by

$$P_{FA}=Q_{\chi_2^2}(\gamma) \quad (22)$$

$$P_D=Q_{\chi_2'^2(\lambda)}(\gamma) \quad (23)$$

The GLRT becomes

$$T(x)=2T_F(x)=$$

$$\frac{2}{N\hat{\sigma}_1^2}\left|\sum_{n=0}^{N-1}x[n]C(n,n_\tau,f_d)\exp(-j2\pi(f_{IF}+f_d)nT_s)\right|^2>\gamma \quad (24)$$

where γ is the threshold of (24).

The noncentrality parameter λ is

$$\lambda=2\lambda_F=\frac{NA^2}{\sigma^2}=2(S/N_0)T_I \quad (25)$$

For the $\lambda=1$ example, the PDF of $T(\mathbf{x})$ for both central and noncentral chi-squared distributions are shown in Fig. 2.

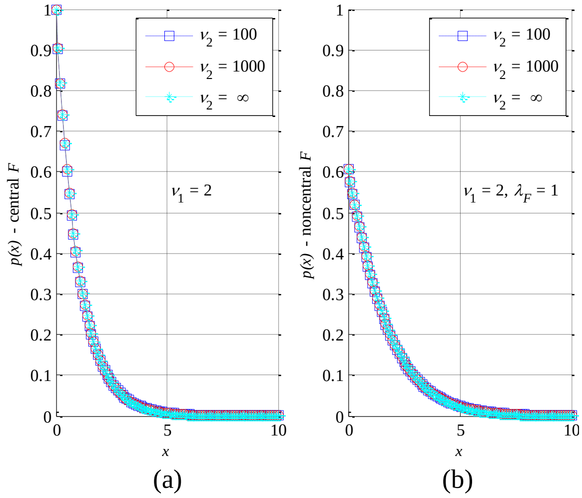

Figure 1: (a) PDF for central F random variables for different v_2 under $v_1=2$; (b) PDF for noncentral F random variables for different v_2 under $v_1=2$ and $\lambda_F=1$

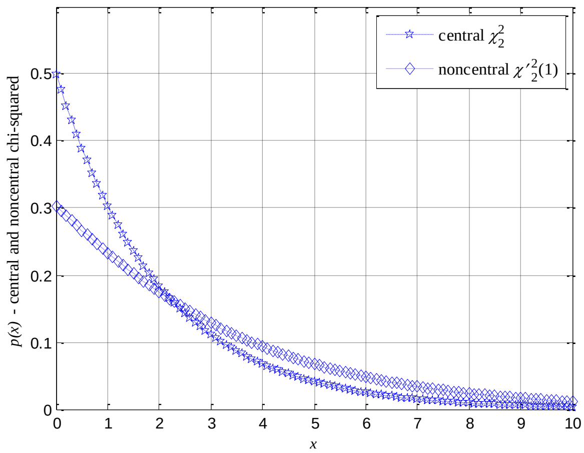

Figure 2: PDF for central and noncentral chi-squared random variables, χ_2^2 and $\chi_2'^2(1)$

3.2 If code phase and Doppler shift are unknown

Secondly, we take consideration of f_d and n_τ besides A,ϕ,σ^2, and rewrite $T(\mathbf{x})$ as $T_{n_\tau,f_d}(\mathbf{x})$, then the question becomes

$$\max_{f_d,n_\tau} T_{n_\tau,f_d}(\mathbf{x})>\gamma \quad (26)$$

which is shown in Fig. 3. Remember that the threshold γ here is different from the one used before.

The probability of false alarm follows as

$$P_{FA}=\Pr\left\{\max_{f_d,n_\tau} T_{n_\tau,f_d}(\mathbf{x})>\gamma;\mathcal{H}_0\right\} \quad (27)$$

The exact distribution of the test statistic $\max_{f_d, n_\tau} T_{n_\tau, f_d}(\mathbf{x})$ is difficult to obtain since there is dependence between $T_{n_\tau, f_d}(\mathbf{x})$ across the Doppler frequency and PRN code phase search space. One overly-conservative method to decide P_{FA} is via the union bound by assuming all the $T_{n_\tau, f_d}(\mathbf{x})$ are independent (Winternitz et al., 2004). Thus, we have

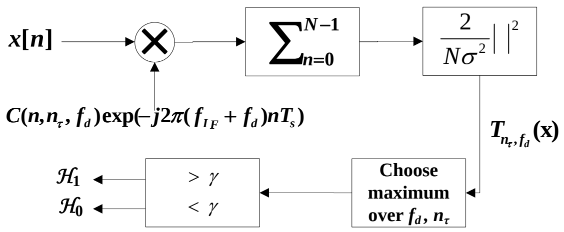

Figure 3: Hypothesis testing analysis for GPS signal detection

$$P_{FA} = 1 - \Pr\left\{\max_{f_d, n_\tau} T_{n_\tau, f_d}(\mathbf{x}) < \gamma; \mathcal{H}_0\right\} = 1 - \Pr\left\{\bigcap_{f_d, n_\tau} T_{n_\tau, f_d}(\mathbf{x}) < \gamma; \mathcal{H}_0\right\}$$
$$= 1 - \prod_{f_d, n_\tau} \Pr\left\{T_{n_\tau, f_d}(\mathbf{x}) < \gamma; \mathcal{H}_0\right\}$$

And $\Pr\left\{T_{n_\tau, f_d}(\mathbf{x}) < \gamma; \mathcal{H}_0\right\} = \int_0^\gamma 0.5\exp(-u/2)du = 1 - \exp(-\gamma/2)$,

Hence $P_{FA} = 1 - \prod_{f_d, n_\tau}[1 - \exp(-\gamma/2)] = 1 - [1 - \exp(-\gamma/2)]^{N_{search}}$.

Note that for a small P_{FA}, we have $\exp(-\gamma/2) << 1$, so using $(1-x)^N \approx 1 - Nx$, for $x << 1$, we get

$$P_{FA} \approx N_{search}\exp(-\gamma/2) = N_{search}P_{FA}(cell) \qquad (28)$$

where

$$P_{FA}(cell) = \exp(-\gamma/2) \qquad (29)$$

is the probability of false alarm if we only examine one cell. Hence P_{FA} increases approximately linearly with the number of cells examined.

A less conservative method (Winternitz et al., 2004; Psiaki, 2001) to decide P_{FA} is via the union bound by only considering $T_{n_\tau, f_d}(\mathbf{x})$ which are mutually independent. For the C/A code phase dimension, we can consider this way. The C/A code has 1023 chips in 1ms and then repeats itself. It is nearly uncorrelated with itself, except for between ±1 chips. Normally, the sampling frequency f_s is larger than twice of C/A code rate 1.23MHz, so obviously we have $N_{ms} > 1023$ (Spiker, 1996; Ward et al., 2006). The less conservative method can be applied to use 1023 instead of N_{ms} as the independent searching cells of C/A code phase dimension. For the Doppler frequency dimension, we can consider this way. The frequency searching step is closely related to the length of the data used in the acquisition (Tsui, 2005). When the input signal and the locally generated complex signal are off by 1 cycle there is no correlation. When the two signals are off less than 1 cycle there is partial correlation (Tsui, 2005). Normally, it is chosen that the maximum frequency separation allowed between the two signals is 0.5 cycle. For GPS signal of coherent integration time T_I (in second), a $1/T_I$ (in Hz) signal will change 1 cycle in T_I. In order to keep the maximum frequency separation at 0.5 cycle in T_I, the frequency step should be $1/T_I$. Under this condition, the number of searching frequency cells in T_I is defined as N_{f_d}. Thus, we have $N'_{search} = 1023N_{f_d}$ and then

$$P_{FA} \approx N'_{search}\exp(-\gamma/2) = N'_{search}P_{FA}(cell) \qquad (30)$$

It will give a very good approximation to the desired threshold. In the rest of the paper, this less conservative method is applied.

To find the probability of detection we first define a detection as a threshold crossing in the correct code phase and Doppler frequency cell. Hence, P_D is defined as the probability that the maximum of the spectrogram occurs in the correct cell, i.e., $n_\tau = n_{\tau c}$, $f_d = f_{dc}$, when a signal is present. With this definition we have

$$\begin{aligned} P_D &= \Pr\left\{T_{n_{\tau c}, f_{dc}}(\mathbf{x}) > \gamma; \mathcal{H}_1\right\} \\ &= Q_{\chi_2'^2(\lambda)}(\gamma) = Q_{\chi_2'^2(\lambda)}\left(2\ln\left(N'_{search}/P_{FA}\right)\right) \end{aligned} \qquad (31)$$

where $\lambda = 2(S/N_0)T_I$.

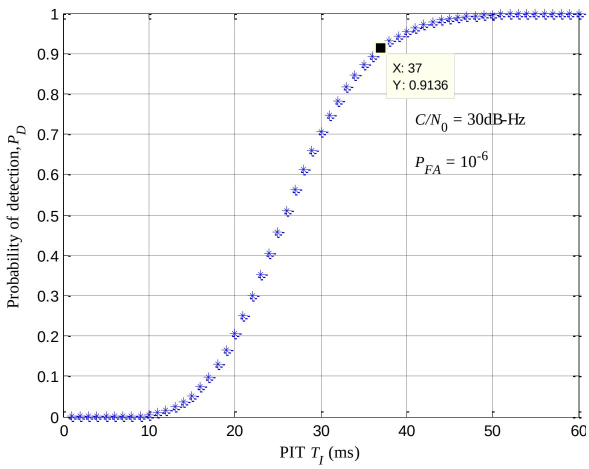

Figure 4: P_D versus T_I under $C/N_0 = 30$dB-Hz and $P_{FA} = 10^{-6}$

As an example, suppose $C/N_0 = 30\text{dB}-\text{Hz}$, $f_s = 2.048\text{MHz}$, and $f_d \subset$ (-5, 5)kHz, so $\lambda = 2\times10^3 T_I$ and $N'_{search} = 1023\times10^4 \times T_I$. Given $P_{FA} = 10^{-6}$, the relation between T_I and P_D is plotted in Fig. 4. The threshold is set based on P_{FA}. We can see that to obtain $P_D = 0.9$, $T_I = 37\text{ms}$ can be applied, under the conditions that the data bits and bit edge in this 37ms is known as a priori.

4. Considering Noncoherent Integration

Coherent integration over T_I is the first step in any acquisition method to find the GPS signals, because sometimes using T_I of data cannot detect a weak signal while only increase T_I requires many more operations. One way to process more data is through noncoherent integration. Typically a set of long input data is divided into N_{T_I} blocks with PIT time T_I, coherent integration is performed on all the blocks. After the coherent integration, the output at every frequency and code delay is complex and can be put into amplitude form. The amplitude from all the coherent integration of the same frequency and code delay are summed, known as noncoherent integration. As a result, the weak signal will be enhanced, leading to a higher signal to noise ratio.

Suppose we have N_{T_I} blocks of data, $x_m[n]$, where $m = 0,1,\ldots,N_{T_I}-1$; $n = mN, mN+1,\ldots,mN+N-1$. The signal acquisition measurement model for a GPS receiver is illustrated in Fig. 5. After each of GLRT $T_{n_\tau,f_d,m}(\mathbf{x})$ is calculated, we do N_{T_I} noncoherent integration to get $T_{n_\tau,f_d}(\mathbf{x})$ before choosing the maximum over n_τ and f_d. The maximum is then compared against the threshold γ to determine if the signal is present or not. Remember that the γ here is different from the ones used before.

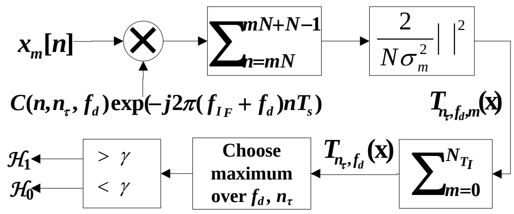

Figure 5: Hypothesis testing analysis for GPS signal detection with noncoherent integration

Using Equation (24), the GLRT $T_{n_\tau,f_d,m}(\mathbf{x})$ and $T_{n_\tau,f_d}(\mathbf{x})$ are expressed as

$$T_{n_\tau,f_d,m}(\mathbf{x}) = \frac{2}{N\hat{\sigma}_m^2}\left|\sum_{n=mN}^{mN+N-1} x_m[n]C(n,n_\tau,f_d)\exp\left(-j2\pi\left(f_{IF}+f_d\right)nT_s\right)\right|^2 \tag{32}$$

$$T_{n_\tau,f_d}(\mathbf{x}) = \sum_{m=1}^{N_{T_I}} T_{n_\tau,f_d,m}(\mathbf{x}) \tag{33}$$

We decide $\mathcal{H}_1$ if

$$\max_{f_d,n_\tau} T_{n_\tau,f_d}(\mathbf{x}) > \gamma \tag{34}$$

If $T_{n_\tau,f_d,m}(\mathbf{x})$ has a central or noncentral chi-squared distribution of 2 degrees, $T_{n_\tau,f_d}(\mathbf{x})$ has a central or noncentral chi-squared distribution of $2N_{T_I}$ degrees (Kay, 1993-1).

Now we continue to determine the detection performance of this testing. First we are going to determine P_{FA}. Under $\mathcal{H}_0$, the test statistic is as follows

$$T_{n_\tau,f_d}(\mathbf{x}) \sim \chi^2_{2N_{T_I}} \tag{35}$$

The general probability density function of $y = T_{n_\tau,f_d}(\mathbf{x})$ under hypothesis $\mathcal{H}_0$, as a function of N_{T_I}, is as follows

$$p(y;H_0) = \frac{1}{2^{N_{T_I}}\left(N_{T_I}-1\right)!} y^{N_{T_I}-1} e^{-y/2}; y \ge 0 \tag{36}$$

which describes a chi-squared density function with $2N_{T_I}$ degrees of freedom.

By considering $T_{n_\tau,f_d}(\mathbf{x})$ which are mutually independent, the corresponding probability of false alarm for a threshold γ is then as follows

$$P_{FA} = \Pr\left\{\max_{f_d,n_\tau} T_{n_\tau,f_d}(\mathbf{x}) > \gamma; \mathcal{H}_0\right\} \approx N'_{search} Q_{\chi^2_{2N_{T_I}}}(\gamma) \tag{37}$$

which is a function of the threshold γ, the noncoherent integration number N_{T_I} and the number of searching cells N'_{search}. For a given threshold γ and sum N_{T_I}, $Q_{\chi^2_{2N_{T_I}}}(\gamma) = \int_\gamma^\infty p(y;\mathcal{H}_0)dy$ can be determined from tables, or from the following incomplete gamma function

$$Q_{\chi^2_{2N_{T_I}}}(\gamma) = e^{-\frac{\gamma}{2}} \sum_{k=0}^{N_{T_I}-1} \frac{1}{k!}\left(\frac{\gamma}{2}\right)^k \tag{38}$$

P_D is defined as the probability that the maximum of the spectrogram occurs in the correct cell, i.e., $n_\tau = n_{\tau c}$, $f_d = f_{dc}$, when a signal is present. With this definition we have

$$P_D = \Pr\left\{T_{n_{\tau c}, f_{dc}}(\mathbf{x}) > \gamma; \mathcal{H}_1\right\} = Q_{\chi'^2_{2N_{T_I}}(\lambda)}(\gamma) \quad (39)$$

where

$$\lambda = 2(S/N_0)T_I N_{T_I} \quad (40)$$

The probability density function of $y = T_{n_{\tau c} f_{dc}}(\mathbf{x})$, as a function of N_{T_I} , is as follows

$$p(y;\mathcal{H}_1) = \frac{1}{2}\left(\frac{y}{\lambda}\right)^{\frac{1}{2}(N_{T_I}-1)} e^{-\frac{1}{2}(y+\lambda)} I_{N_{T_I}-1}\left(\sqrt{\lambda y}\right), y \geq 0 \quad (41)$$

where $I_r(u)$ is the modified Bessel function of the first kind with order r . The equation describes a noncentral chi-squared density function with $2N_{T_I}$ degree of freedom. The corresponding probability of detection for the threshold γ is then calculated as

$$P_D = \int_\gamma^\infty p(y;\mathcal{H}_1)dy \quad (42)$$

Usually, the desired probability of false alarm is a given. Thus the procedure is first to use equations (37) and (38) to solve for the threshold γ for that desired probability of false alarm. The only way that the threshold γ can be determined for a desired P_{FA} is to do it iteratively via trail-and-error, a method such as the Newton-Rhapson method (Dierendonck, 1996). And then evaluate the performance of the detector as a function of C/N_0 . If it doesn't perform well enough, either N_{T_I} or T_I , or both, will have to be increased, which effectively slows down the search rate. Increasing T_I (decreasing the predetection bandwidth) is more effective, but not always possible because of Doppler frequency stability, or because of data bit edge occurrence and local oscillator stability.

5. Examples

Here are some examples to illustrate the relations among PIT T_I , noncoherent integration number N_{T_I} , threshold γ and C/N_0 to achieve a probability of detection $P_D = 0.9$ for a given probability of false alarm $P_{FA} = 10^{-6}$.

Suppose $C/N_0 = 30\text{dB}-\text{Hz}$, $f_s = 2.048\text{MHz}$, and $f_d \subset (-5,5)\text{kHz}$, so $\lambda = 2\times10^3 T_I N_{T_I}$ and $N'_{search} = 1023\times10000\times T_I$. For the given $P_{FA} = 10^{-6}$, the relations between T_I , N_{T_I} and P_D are plotted in Fig. 6 and the relations between T_I , N_{T_I} and γ are plotted in Fig. 7. We can see that to obtain $P_D = 0.9$, the combinations of T_I (in ms) and N_{T_I} can be {4,16},{6,8},{11,4},{20,2} and {37,1}, or other possible combinations which are not shown in Fig. 6, if the data bits in this T_I are known as a priori. We can see that the curve in Fig. 4 is the same as the curve of $N_{T_I} = 1$ in Fig. 6, this can be easily understood that a coherent integration over T_I can be treated as a noncoherent integration over T_I with noncoherent integration number $N_{T_I} = 1$. Considering weak GPS signals, we suppose $C/N_0 = 20\text{dB}-\text{Hz}$ and the other parameters are unchanged. The relations between T_I , N_{T_I} and P_D are plotted in Fig. 8 and the relations between T_I , N_{T_I} and γ are plotted in Fig. 9. We can see that to obtain $P_D = 0.9$, the combinations of T_I (in ms) and N_{T_I} can be {37,16},{64,8},{113,4},{207,2} and {391,1}, or other possible combinations which are not shown in Fig. 8. Table 1 shows the combinations of (N_{T_I} , T_I , γ) for both $C/N_0 =$ 30 and 20dB-Hz under $P_{FA} = 10^{-6}$ and $P_D \geq 0.9$.

Table 1: The Combinations of (N_{T_I} , T_I , γ) for $C/N_0 = 30$ and 20dB-Hz under $P_{FA} = 10^{-6}$ and $P_D \geq 0.9$

C/N_0 (dB-Hz)	N_{T_I}	T_I (ms)	γ
30	1	37	54
	2	20	59
	4	11	69
	8	6	86
	16	4	116
20	1	391	59
	2	207	64
	4	113	74
	8	64	92
	16	37	122

Because the threshold has nothing to do with C/N_0 , Fig. 7 is the same as Fig. 9 for T_I from 1 to 60ms. But the threshold changes as T_I changes, while T_I is related to coherent integration sample number N . But in Dierendonck (1996), it has declared that increasing N does not change the threshold. This is only correct if we have a priori information of exact code delay and

Doppler frequency shift. In actual situations, we only have a range of code delay and Doppler frequency shift and then we have to search for all the possible combinations to detect the correct values. Hence increasing N does change the threshold.

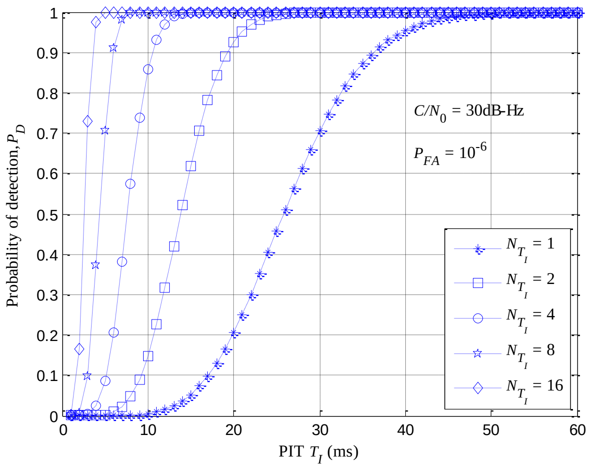

Figure 6: P_D versus T_I for different N_{T_I} under $C/N_0 = 30$dB-Hz and $P_{FA} = 10^{-6}$

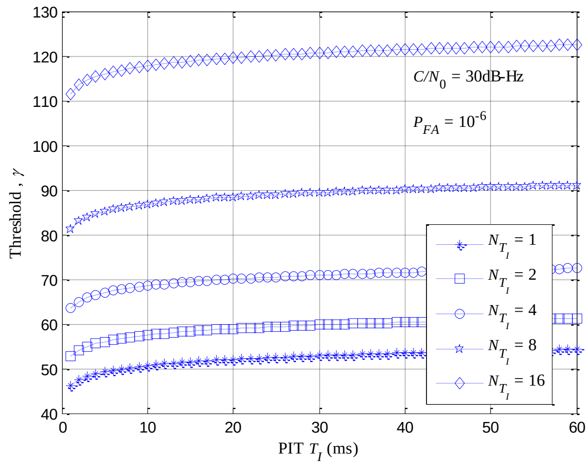

Figure 7: γ versus T_I for different N_{T_I} under $C/N_0 = 30$dB-Hz and $P_{FA} = 10^{-6}$

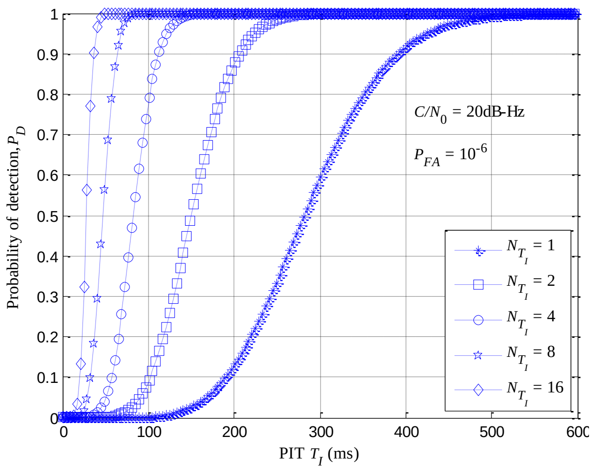

Figure 8: P_D versus T_I for different N_{T_I} under $C/N_0 = 20$dB-Hz and $P_{FA} = 10^{-6}$

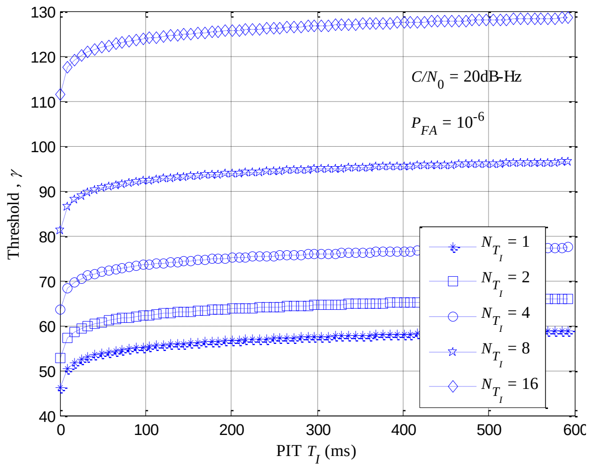

Figure 9: γ versus T_I for different N_{T_I} under $C/N_0 = 20$dB-Hz and $P_{FA} = 10^{-6}$

If we have a priori information of exact code delay and Doppler frequency shift, then $N_{search} = 1$, using γ_1 instead of γ, we have

$$P_{FA} = Q_{\chi^2_{2N_{T_I}}}(\gamma_1) \tag{43}$$

Given $P_{FA} = 10^{-6}$,for $N_{T_I} = \{1,2,4,8,16\}$, the corresponding γ_1 are calculated to be γ_1={27.63, 33.38, 42.70, 58.32, 85.23}. The differences of the threshold between γ in Fig. 9 and γ_1 versus T_I are shown in Fig. 10. The differences at T_I = 1, 20, and 600ms for each N_{T_I} are shown in Table 2. They are increasing as T_I increases. It is apparent that N'_{search} cannot be simplified as $N'_{search} = 1$.

Table 2: Threshold Differences $\gamma - \gamma_1$ for $T_I = 1$, 20, 600ms

	$\gamma - \gamma_1$				
N_{T_I}	1	2	4	8	16
$T_I = 1$ms	19	20	21	23	27
$T_I = 20$ms	25	26	28	31	35
$T_I = 600$ms	32	33	35	39	44

Now we decide the relation between C/N_0 and the probability of detection P_D under different T_I and N_{T_I} for the given $P_{FA}=10^{-6}$. Set $N_{T_I}=16$, for different T_I, P_D versus C/N_0 are plotted in Fig. 11. We can tell that to obtain a probability of detection $P_D=0.9$, the minimum detectable C/N_0 are {23, 20, 18, 17, 16} (in dB-Hz) at $T_I=\{20,\ 40,\ 60,\ 80,\ 100\}$(in ms) separately. The results are shown in Table 3. Set T_I = 40ms, for different N_{T_I}, the probability of detection versus C/N_0 are plotted in Fig. 12. We can tell that to obtain a probability of detection $P_D=0.9$, the minimum detectable C/N_0 are {22, 20, 19, 18, 17} (in dB-Hz) at N_{T_I} = {8, 16, 24, 32, 40} separately. The results are shown in Table 4.

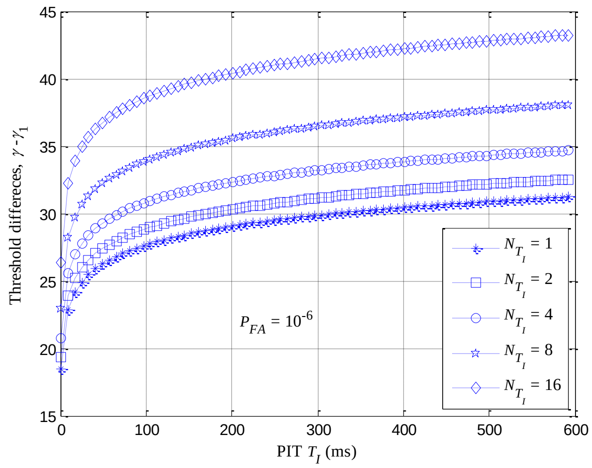

Figure 10: Threshold differences $\gamma-\gamma_1$ versus T_I for different N_{T_I} under $P_{FA}=10^{-6}$

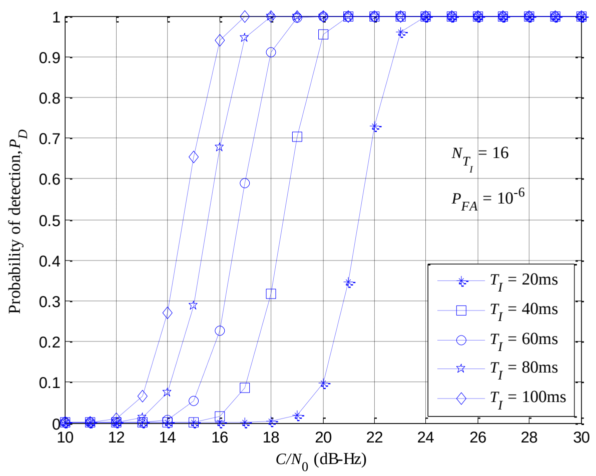

Figure 11: P_D versus C/N_0 for different T_I under $N_{T_I}=16$ and $P_{FA}=10^{-6}$

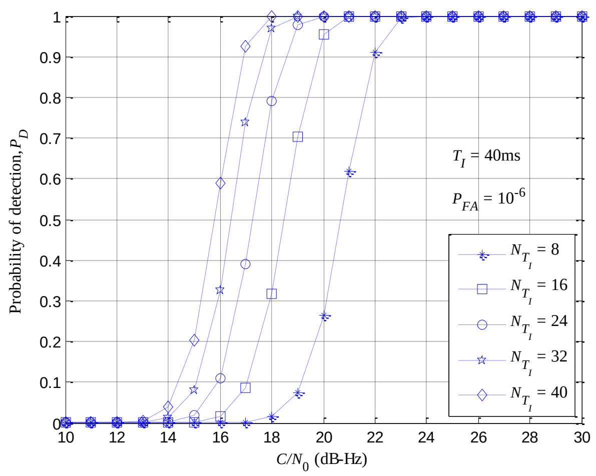

Figure 12: P_D versus C/N_0 for different N_{T_I} under T_I =40ms and $P_{FA}=10^{-6}$

Table 3: The Minimum Detectable C/N_0 for Different T_I under $N_{T_I}=16$

N_{T_I}	16				
T_I (ms)	20	40	60	80	100
C/N_0 (dB-Hz)	23	20	18	17	16

Table 4: The Minimum Detectable C/N_0 for Different N_{T_I} under T_I =40ms

T_I (ms)	40				
N_{T_I}	8	16	24	32	40
C/N_0 (dB-Hz)	22	20	19	18	17

6. Summaries and Conclusions

In this paper, GPS signal detection using hypothesis testing analysis are given, using the model of IF GPS signal of one satellite in WGN. The GLRT approach is applied to detect the GPS signal under hypotheses $\mathcal{H}_1$ and $\mathcal{H}_0$.

Based on detection theory, it has proved that in fact the GPS test statistic follows central or noncentral F distribution because the power of the WGN is unknown, as well as that the signal is also uncertain. But the statistic is nearly identical to central or noncentral chi-squared distribution because the processing samples are large enough to be considered as infinite in GPS acquisition algorithms. The proof is shown in Fig. 1

where the PDFs for central and noncentral F distribution are nearly the same, respectively, when the denominators are large.

It has also proved that the probability of false alarm, the probability of detection and the threshold are affected largely when the hypothesis testing refers to the full PRN code phase and Doppler frequency search space cells instead of to each individual cell. The performance of the test statistic is also given with combining the noncoherent integration to increase the processing gain.

Given the probability of false alarm to achieve a desired probability of detection, examples are illustrated to determine the relations among the threshold, the PIT, the number of noncoherent integration and signal to noise ratio. For the given $P_{FA} = 10^{-6}$, to obtain $P_D = 0.9$ under $C/N_0 = 30$dB-Hz, the combinations of T_I (in ms) and N_{T_I} can be {4,16},{6,8},{11,4},{20,2} and {37,1}; under $C/N_0 = 20$dB-Hz, the combinations of T_I (in ms) and N_{T_I} can be {37,16},{64,8},{113,4},{207,2} and {391,1}. The threshold is related to N_{T_I} and T_I, but independent of C/N_0. The differences of the threshold between using the less conservative method and only considering one individual cell are significant with values {19, 20, 21, 23, 27} at $T_I = 1$ms for N_{T_I} ={1, 2, 4, 8, 16}, respectively, whilethey are increasing as T_I increases. So it is apparent that N'_{search} cannot be simplified as $N'_{search} = 1$. For the given $P_{FA} = 10^{-6}$, to obtain $P_D = 0.9$ under $N_{T_I} = 16$, the minimum detectable C/N_0 are {23, 20, 18, 17, 16} (in dB-Hz) at T_I = {20, 40, 60, 80, 100}(in ms) separately; under $T_I = 40$ms, the minimum detectable C/N_0 are {22, 20, 19, 18, 17} (in dB-Hz) at N_{T_I} = {8, 16, 24, 32, 40} separately.

In this paper, the effect of data bits and bit edges in PIT T_I is neglected temporarily to simplify the GPS signal detection problem. But for weak signals, how to decide the data bit sequence and bit edges will make the problem more important and complicated. More work about this aspect has been done by the authors.

Appendix

Theorem 7.1 (GLRT for Classical Linear Model) Assume the data have the form $\mathbf{x} = \mathbf{H}\boldsymbol{\theta} + \mathbf{w}$, where $\mathbf{H}$ is a known $N \times p(N > p)$ observation matrix of rank p, $\boldsymbol{\theta}$ is a $p \times 1$ vector of parameters, and $\mathbf{w}$ is an $N \times 1$ noise vector with PDF $\mathcal{N}(0, \sigma^2\mathbf{I})$. The GLRT for the hypothesis testing problem

$$\begin{aligned} \mathcal{H}_0 &: \mathbf{A}\boldsymbol{\theta} = \mathbf{b} \\ \mathcal{H}_1 &: \mathbf{A}\boldsymbol{\theta} \neq \mathbf{b} \end{aligned} \quad \text{(A.1)}$$

where $\mathbf{A}$ is an $r \times p(r \le p)$ matrix of rank r, $\mathbf{b}$ is an $r \times 1$ vector, and $\mathbf{A}\boldsymbol{\theta} = \mathbf{b}$ is a consistent set of linear equations, is to decide $\mathcal{H}_1$ if

$$T(x) = \frac{\left(\mathbf{A}\hat{\boldsymbol{\theta}}_1 - \mathbf{b}\right)^T \left(\mathbf{A}\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{A}^T\right)^{-1}\left(\mathbf{A}\hat{\boldsymbol{\theta}}_1 - \mathbf{b}\right)}{\sigma^2} > \gamma' \quad \text{(A.2)}$$

where

$$\hat{\boldsymbol{\theta}}_1 = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{x} \quad \text{(A.3)}$$

is the MLE of $\boldsymbol{\theta}$ under $\mathcal{H}_1$. The exact detection performance is given by

$$\begin{aligned} P_{FA} &= Q_{\chi_r^2}(\gamma') \\ P_D &= Q_{\chi'^2_r(\lambda)}(\gamma') \end{aligned} \quad \text{(A.4)}$$

where the noncentrality parameters is

$$\lambda = \frac{\left(\mathbf{A}\boldsymbol{\theta}_1 - \mathbf{b}\right)^T \left(\mathbf{A}\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{A}^T\right)^{-1}\left(\mathbf{A}\boldsymbol{\theta}_1 - \mathbf{b}\right)}{\sigma^2} \quad \text{(A.5)}$$

Theorem 9.1 (GLRT for Classical Linear Model — σ^2 Unknown) Assume the data have the form $\mathbf{x} = \mathbf{H}\boldsymbol{\theta} + \mathbf{w}$, where $\mathbf{H}$ is a known $N \times p(N > p)$ observation matrix of rank p, $\boldsymbol{\theta}$ is a $p \times 1$ vector of parameters, and $\mathbf{w}$ is an $N \times 1$ noise vector with PDF $\mathcal{N}(0, \sigma^2\mathbf{I})$. The GLRT for the hypothesis testing problem

$$\begin{aligned} \mathcal{H}_0 &: \mathbf{A}\boldsymbol{\theta} = \mathbf{b}, \sigma^2 > 0 \\ \mathcal{H}_1 &: \mathbf{A}\boldsymbol{\theta} \neq \mathbf{b}, \sigma^2 > 0 \end{aligned} \quad \text{(A.6)}$$

where $\mathbf{A}$ is an $r \times p(r \le p)$ matrix of rank r, $\mathbf{b}$ is an $r \times 1$ vector, and $\mathbf{A}\boldsymbol{\theta} = \mathbf{b}$ is a consistent set of linear equations, is to decide $\mathcal{H}_1$ if

$$T(x) = \frac{(N-p)}{r} \frac{\left(\mathbf{A}\hat{\boldsymbol{\theta}}_1 - \mathbf{b}\right)^T \left(\mathbf{A}\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{A}^T\right)^{-1}\left(\mathbf{A}\hat{\boldsymbol{\theta}}_1 - \mathbf{b}\right)}{\mathbf{x}^T\left(\mathbf{I} - \mathbf{H}\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\right)\mathbf{x}} > \gamma' \quad \text{(A.7)}$$

where

$$\hat{\boldsymbol{\theta}}_1 = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{x} \quad \text{(A.8)}$$

is the MLE of $\boldsymbol{\theta}$ under $\mathcal{H}_1$ or the unrestricted MLE. The exact detection performance (holds for finite data records) is given by

$$P_{FA} = Q_{F_{r,\,N-p}}(\gamma') \qquad \text{(A.9)}$$
$$P_D = Q_{F'_{r,\,N-p}}(\gamma')$$

where $F_{r,\,N-p}$ denotes an F distribution with r numerator degrees of freedom and $N-p$ denominator degrees of freedom. And $F'_{r,\,N-p}(\lambda)$ denotes a noncentral F distribution with r numerator degrees of freedom, $N-p$ denominator degrees of freedom and noncentrality parameter λ which is given by

$$\lambda = \frac{(\mathbf{A}\boldsymbol{\theta}_1 - \mathbf{b})^T \left(\mathbf{A}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{A}^T\right)^{-1}(\mathbf{A}\boldsymbol{\theta}_1 - \mathbf{b})}{\sigma^2} \qquad \text{(A.10)}$$

where $\boldsymbol{\theta}_1$ is the true value of $\boldsymbol{\theta}$ under $\mathcal{H}_1$.

The unbiased estimator of σ^2 is

$$\hat{\sigma}_1^2 = \frac{1}{N-p}\mathbf{x}^T\left(\mathbf{I} - \mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\right)\mathbf{x} \qquad \text{(A.11)}$$

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