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Observability Analysis of Non-Holonomic Constraints for Land-Vehicle Navigation Systems

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Abstract

Over the past decades, the integration of a MEMS-based (Micro-Electro Mechanical Systems) Inertial Measuring Unit (IMU) with a GNSS receiver-chip has become commonly used navigation techniques by virtue of their advantages such as small sized, light weight, with low power consumption, and have extremely low cost. To provide accurate and reliable positioning solutions with a low-cost GNSS/MEMS INS system, it is valuable to introduce specific auxiliary information that can improve the navigation performance without adding extra hardware costs. The auxiliary information is especially useful during GNSS outage periods or when the vehicle is moving with low dynamics (e.g. no change of attitude and accelerations) which lead to the poor observability of the GNSS/INS navigation system. For LVN applications, Non-Holonomic Constraints (NHC) is one of the most common types of auxiliary information.

This paper focuses on studying the contributions of the NHC from the perspective of observability, which provides a deeper insight and shows how the NHC improves the navigation solutions. Considering several typical vehicle dynamics, it is also clear to see the effects of the NHC to the inertial navigation under different situations. Both theoretical analysis and simulation tests have shown that the contributions of the NHC to the estimation of a certain state depend on both the current vehicle dynamic and the relative error magnitude of this state compared to the coupled state under the current vehicle dynamic; both the accelerating and turning motions can enhance the contributions of the NHC to the estimation of both the yaw and the pitch, and such contributions will be stronger with a higher vehicle speed; the NHC has significant effects on controlling the roll in all motion status. Furthermore, the effects of the NHC on the estimation of the biases of both gyroscopes and accelerometers are also analyzed. The outcomes of this paper show that the proposed observability analysis is beneficial to the utilization of NHC or other priori information in low-cost navigation systems.

Keywords: GNSS/INS, non-holonomic constraints, observability, inertial measurement unit (IMU), inertial navigation

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1. Introduction

GNSS (Global Navigation Satellite System) and INS (Inertial Navigation Systems) have different advantages and can be integrated to provide a variety of navigation information which is precise, reliable, and with high data rate (El-Sheimy 2006). A GNSS receiver is mainly used to provide position of the vehicle in ideal conditions, but has certain limitations in urban areas (e.g., city downtown), inside tunnels and under heavy tree canopies. An inertial navigation system measures the specific forces and angular rates by accelerometers and gyroscopes (gyros) and determines the motion of a body with respect to an inertial frame of reference (Titterton and Weston 1997). By virtue of its advantages such as high update rates as well as high precision and reliable measurements over a short time period, INS has always been presented as a valuable system in many Land-Vehicle Navigation (LVN) applications. Especially during the past few decades, advances in Micro-Electro-Mechanical Systems (MEMS) technology combined with the miniaturization of electronics (Karumuri *et al* 2011), have made it possible to produce chip-based inertial sensors for use in measuring angular rate and acceleration. These MEMS chips have become ideal candidates for various applications because they are small, light weight, low power and are extremely low-cost and reliable (El-Sheimy 2006). At the same time, the cost reduction of GNSS receiver has also promoted the development of low-cost navigation techniques. The integration of a MEMS-based Inertial Measurement Unit (IMU) with a GNSS receiver-chip provides a navigation system that has several advantages over each individual system. In such integration, the GNSS-derived positions and velocities are updating the MEMS sensors through a Kalman Filter (KF) while the IMU is used to provide the navigation information during GNSS signal outages and for fast GNSS signal reacquisition (Niu *et al* 2007).

GNSS/MEMS INS integrated systems have become one of the most commonly used navigation techniques.

However, the performances of the MEMS sensors are highly dependent on the environmental conditions such as temperature variations (Aggarwal *et al* 2008). During the GNSS outages in challenging environments such as in urban canyons, a MEMS INS will lose its accuracy quickly due to the large bias instability and noise of low-cost MEMS sensors. The accuracy limitation of low-end navigation techniques (e.g. MEMS IMU, GNSS chips etc.) becomes one of the obstacles for their development and applications.

Therefore, special considerations are required to provide accurate positioning solutions with a GNSS/MEMS INS system. One direct method is adding other low frequency absolute sensors (e.g., GNSS dual antenna systems, wheel and steering encodes, magnetic compass, etc), so as to bound the increasing positioning errors associated with the high frequency inertial sensors (Skog and Händel 2009). This kind of method is quite effective but relies on additional sensors, which is not affordable for many low-end land-based navigation systems.

From the perspective of the navigation system designers, there are also several approaches especially valuable for the low-end navigation systems due to their independence of hardware costs. The first approach is to improve the navigation algorithms used. Examples of such algorithms implement an unscented KF (Shin 2004) or use neural networks (Chiang 2004). Another approach is to improve the stochastic modeling of the MEMS inertial sensors (Nassar and El-Sheimy 2005a). Moreover, a third method is to enhance the quality of the MEMS data through techniques such as de-noising (Nassar and El-Sheimy 2005b). The fourth and commonly used approach is to introduce priori information about the navigation systems (e.g. control inputs, vehicle dynamic models, kinematic constraints, the road information etc.), which can play significant roles in generating information and reducing estimation uncertainty. One application of this approach is to introduce auxiliary update information (or constraints) to the MEMS sensors to improve KF error estimation and compensation especially during GNSS signal blockages (Dissanayake *et al* 2001, Syed *et al* 2008, Niu *et al* 2010). In this paper, this approach will be investigated in detail. For LVN applications, the most common types of auxiliary information used include Non-Holonomic Constraint (NHC), Zero-Velocity Update (ZUPT), Zero Integrated Heading Rate (ZIHR), etc. (Shin 2005). This paper will focus on the NHC, which is the most typical one among these constraints.

According to the observability analysis of the errors in a GNSS/INS integrated navigation system, all the errors can be made observable by maneuverings (Hong *et al* 2005). However, the land-based vehicles do not have sufficient maneuvers all the time, which will cause observability problems. During most of the time, a car moves straightforward with small velocity variations. Under this condition, the yaw is unobservable or weakly observable even when there are GNSS updates (Porat and Bar-Itzhack 1981). Once the GNSS signal was blocked, the yaw would diverge even faster. The degradation of the yaw angle will affect the position estimates directly. Therefore, it is beneficial to use NHC, which refer to the fact that unless the vehicle jumps off or slides on the ground, the velocity of the vehicle in the plane perpendicular to the forward direction is almost zero. It had already been shown by real tests that both the position errors and the attitude errors were reduced significantly when the NHC were used (Niu *et al* 2007).

Although it is widely known that the NHC can improve the navigation performance (Dissanayake *et al* 2001, Niu *et al* 2007), there is relatively less theoretical analysis to their contributions. Observability analysis is an effective tool to study the ability of estimating the states of a system. It provides a deeper insight into the navigation algorithms.

This paper tends to analyze the contributions of the NHC to the estimated states from the perspective of observability. It will show how the NHC work in the navigation algorithms. Combining with several typical vehicle dynamics, it is also clear to see the effect of NHC under different situations. Then it is beneficial to grasp this widely used constraint and maximize its effects, so as to promote the better utilization of NHC or other priori information.

Comparing with the previous works, this paper focuses on analyzing the contributions of the NHC to the specific estimated states from the perspective of observability. In the rest of this paper, the navigation algorithms, observability analysis and the simulation method will be explained in turn, followed by the simulation results and conclusions.

2. Navigation Algorithm

As shown in Fig.1, the NHC is added into a loosely-coupled architecture of GNSS/INS integration algorithm. Here the reference frame to implement the navigation reference frame is the local-level-frame (n-frame, North-East-Down), and the corresponding IMU body frame (b-frame) is Forward-Right-Down. The GNSS measurements updates are not used, so as to focus on the evaluation to the contributions of the NHC.

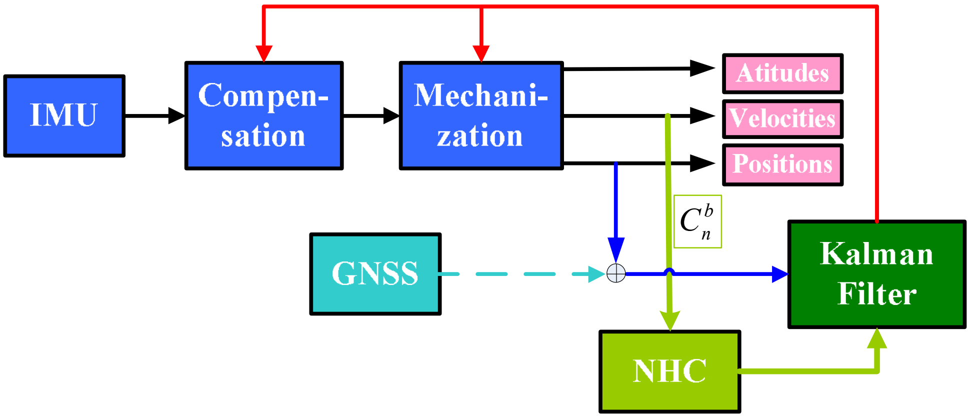

Fig. 1. Architecture of the navigation algorithm

A 15-state Kalman Filter (KF) is designed with the state vector given in (1), which can estimate and compensate the sensor biases online (Shin 2005). The bias instabilities of sensors are modeled as the 1st-order Gaussian-Markov process.

$$\mathbf{x} = \begin{bmatrix} \boldsymbol{\delta}\mathbf{r}^n \\ \boldsymbol{\delta}\mathbf{v}^n \\ \Phi^n \\ \mathbf{b}_g \\ \mathbf{b}_a \end{bmatrix} \tag{1}$$

where,

$\boldsymbol{\delta}\mathbf{r}^n$ is position error vector of INS mechanization,

$\boldsymbol{\delta}\mathbf{v}^n$ is velocity error vector of INS mechanization,

Φ^n is attitude error vector of INS mechanization,

$\mathbf{b}_g$ is the bias vector of gyros, and

$\mathbf{b}_a$ is the bias vector of accelerometers.

The NHC is applied as the KF update measurement in the navigation algorithm. As defined in (Bloch *et al* 2005), non-holonomic systems are mechanical systems with constraints on their velocity that are not derivable from position constraints and arise in mechanical systems that have rolling contact or certain kinds of sliding contact. Therefore, in the case of LVN, the NHC refer to the fact that unless the vehicle jumps off the ground or slides on the ground, the velocity of the vehicle in the plane perpendicular to the forward direction is almost zero (Dissanayake *et al* 2001). Since this constraint is specifically related to wheeled vehicles moving on a surface and is a function of the vehicle state, interaction between the vehicle and the terrain, engine vibrations, and the suspension system, it is non-holonomic. Supposing that the IMU is mounted aligned with the vehicle, this constraints can be regarded as zero velocity update (or ZUPT) along the lateral and vertical axis of the vehicle (right and down), i.e.

$$v_y^b \approx 0 \text{ and } v_z^b \approx 0 \tag{2}$$

where v_y^b and v_z^b are the velocity projection in the body frame (i.e. vehicle frame) along cross-track and vertical directions of the vehicle.

The measurement equation of the INS/NHC system can be written as (Shin 2001)

$$\delta\mathbf{v}^b = \mathbf{C}_n^b\delta\mathbf{v}^n - \mathbf{C}_n^b(\mathbf{v}^n\times)\Phi^n \tag{3}$$

where,

$\delta\mathbf{v}^b$ is the velocity error vector in the b-frame (obtained by the NHC, only the second and third rows are used),

$\mathbf{C}_n^b$ is the rotation matrix from the n-frame to the b-frame,

$\delta\mathbf{v}^n$ is the velocity error in the n-frame,

$(\mathbf{v}^n\times)$ is the skew symmetric matrix of the velocity vector in the n-frame, and

Φ^n is the attitude error vector in the n-frame.

3. Observability Analysis

3.1 Observability definitions

Observability describes the ability of estimating the states of a system (Ham and Brown1983). According to (Hong *et al* 2005), for a linear continuous-time system

$$\Sigma : \dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t)$$
$$\mathbf{z}(t) = \mathbf{C}(t)\mathbf{x}(t)$$

where $\mathbf{A}(t)$ and $\mathbf{C}(t)$ denote the $n\times n$ dynamic matrix and the $p\times n$ design matrix, respectively.

The observability analysis can be treated as finding a state vector $\mathbf{x}(t)$ so that

$$\begin{aligned}\mathbf{z}(t) &= \mathbf{N}_0(t)\mathbf{x}(t) = \mathbf{0}\\ \dot{\mathbf{z}}(t) &= \mathbf{N}_1(t)\mathbf{x}(t) = \mathbf{0}\\ &\vdots\\ \overset{(n-1)}{\mathbf{z}}(t) &= \mathbf{N}_{n-1}(t)\mathbf{x}(t) = \mathbf{0}\end{aligned} \tag{4}$$

For a time $t > t_0$, if there is no nonzero state that satisfies the above conditions, then the system is observable at t_0. Otherwise, any nonzero state $\mathbf{x}_u(t)$ would be an unobservable mode of the system. $\mathbf{N}_0(t)$, $\mathbf{N}_1(t)$,..., $\mathbf{N}_{n-1}(t)$ are a sequence of $p \times n$ observability matrices defined by the equation

$$\mathbf{N}_{k+1}(t) = \mathbf{N}_k(t)\mathbf{A}(t) + \frac{d}{dt}\mathbf{N}_k(t),\ k = 0,1,2,...,n-2$$
$$\mathbf{N}_0(t) = \mathbf{C}(t)$$

There are also other definitions and doable methods for observability analysis (Ham and Brown 1983, Bar-Itzhack and Porat 1988, Goshen-Meskin and Bar-Itzhack 1992). In this paper the definition by Hong (2005) is applied to make the analysis.

3.2 Navigation error model in the body frame

Since NHC is the velocity constraints in the b-fame (i.e. body frame), the theoretical analysis is presented in the b-frame to make them simpler. First the navigation equations of an INS/NHC system are introduced. To simplify the analysis, the land-based vehicle is assumed to move on a flat surface and the IMU (b-frame) is aligned with the vehicle frame. Then both the roll and pitch angles of the IMU are treated as zero. In addition, as low-grade inertial sensors are used, the angular rates caused by the earth rotation and by the vehicle motion are both ignored.

Based on the navigation equations illustrated in (Shin 2005) and the simplifications above, the following continuous-time state equations of velocity and attitude in the b-frame are derived. As the NHC is velocity constraints, they do not bring direct constraints on the vehicle position (Dissanayake *et al* 2001). Thus the state equation of position is not introduced.

$$\boldsymbol{\delta}\dot{\mathbf{v}}^b = \mathbf{F}_{vv}\boldsymbol{\delta}\mathbf{v}^b + \mathbf{F}_{v\phi}\Phi^b + \mathbf{b}_a \tag{5}$$

$$\dot{\Phi}^b = \mathbf{F}_{\phi\phi}\Phi^b - \mathbf{b}_g \tag{6}$$

where,

$$\mathbf{F}_{vv} = \mathbf{F}_{\phi\phi} = \begin{bmatrix} 0 & \omega_z & 0 \\ -\omega_z & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\mathbf{F}_{v\phi} = \begin{bmatrix} 0 & g & f_y \\ -g & 0 & -f_x \\ -f_y & f_x & 0 \end{bmatrix}$$

where ω_z is the vertical angular rate of the vehicle (i.e. the vertical component of the angular rate of the b-frame with respect to the n-frame). f_x and f_y are the acceleration of the vehicle in the forward and cross-track directions, respectively. g is the value of the local gravity.

The sensor biases are regarded as random constants in the analysis.

$$\dot{\mathbf{b}}_g = \mathbf{0} \tag{7}$$

$$\dot{\mathbf{b}}_a = \mathbf{0} \tag{8}$$

And the measurement vector is

$$\mathbf{z}(t) = \begin{bmatrix} \delta v_y^b & \delta v_z^b \end{bmatrix}^T \tag{9}$$

where δv_y^b and δv_z^b are the velocity errors along the y-axis and z-axis in the b-frame, respectively.

3.3 Observability analysis of INS/NHC system

From the definitions and the navigation equations above, it is clear to see that the observability of the estimated states is impacted by f_x , f_y , ω_z and their time derivatives. That is, the observability properties depend on the specific force and angular rate of the vehicle. For the various trajectories in reality, the complexity of vehicle maneuvers will lead to the time-varying of the system process equations, which makes the theoretical observability analysis much more complicated. To make the analysis feasible, four typical dynamics of land-based vehicles (i.e. static / uniform linear motion, variable rectilinear motion, uniform circular motion, and variable angular rate) are considered. The contributions of NHC under different motions are investigated respectively. Here the observable analysis focuses on the attitude estimation and the IMU sensor errors, which are the essential parameters for inertial navigation.

Observability under stationary state or uniform linear motions (i.e., $f_x = f_y = \omega_z = 0$):

Under stationary state, the time derivatives of the measurement equations show that the observable terms are

$$\delta v_y,\ \delta v_z,$$
$$b_{ay} - g\phi_x,\ b_{az},$$
$$\text{and } gb_{gx}$$

where b_{gi} and $b_{ai}(i = x, y, z)$ represent the biases of the i-axis gyro and accelerometer, respectively. $\phi_i(i = x, y, z)$ is the attitude error in the b-frame. These notations are applied in the rest part of this paper.

It is clear that the roll error times gravity and the y-axis accelerometer bias error are jointly observable. The state in these two items that has larger error will get more chance to be corrected. The roll can be controlled to the similar level with the y-axis accelerometer bias error divided by the gravity value. Also, the x-axis gyro bias and z-axis accelerometer bias can be estimated well, while the other states are unobservable.

Observability under variable rectilinear motions (i.e. $f_x \neq 0$, $f_y = \omega_z = 0$):

The observable terms are

$$\delta v_y,\ \delta v_z,$$
$$b_{ay} - g\phi_x - f_x\phi_z,\ b_{az} + f_x\phi_y,$$
$$gb_{gx} + f_x b_{gz} - \dot{f}_x\phi_z,\ \ -f_x b_{gy},$$
$$\dot{f}_x b_{gz} - \ddot{f}_x\phi_z,\ \ -\dot{f}_x b_{gy},$$
$$\ldots$$

when the velocity of vehicle changes, both the yaw (ϕ_z) and the pitch (ϕ_y) can be estimated because of the resultant projection of the specific force onto the y-axis or z-axis. Specially, both the yaw and the pitch can be further distinguished from their coupling states when f_x changes. On the contrary, the observability of the roll would be disturbed by the inclusion of the term $-f_x\phi_z$. Similarly, it is possible to enhance the estimates of both the y-axis and z-axis gyro biases, i.e. b_{gy} and b_{gz}.

Observability under uniform circular motions (i.e. $f_x = 0$, $\omega_z = constant$ and $f_y = \omega_z v_x$):

The observable terms are

$$\delta v_y,\ \delta v_z,$$
$$b_{ay} - g\phi_x - \omega_z\delta v_x,\ b_{az} - f_y\phi_x,$$
$$gb_{gx} - \omega_z(b_{ax} + g\phi_y) - \omega_z g\phi_y - \omega_z^2(\delta v_y + v_x\phi_z),$$
$$\omega_z v_x(b_{gx} - \omega_z\phi_y),$$
$$2\omega_z g(b_{gy} + \omega_z\phi_x) - \omega_z^2(b_{ay} - g\phi_x - v_x b_{gz} - \omega_z\delta v_x),$$
$$\omega_z^2 v_x(b_{gy} + \omega_z\phi_x),$$
$$\ldots$$

when the vehicle experiences a uniform circular motion, both the yaw and pitch estimation can be enhanced. However, due to the constant vertical angular rate, the whole observable terms cannot be divided into separate observable states; thus the contributions to the yaw and the pitch also depend on their coupled states and may be obscured. It also shows that the unobservable part of the yaw is coupled with the velocity error along the cross-track direction. Since the coefficient of the yaw error is the speed of the vehicle, the larger the speed, the better the observability of the yaw; On the contrary, the NHC will have little effect under a low vehicle speed or zero speed.

Observability under variable angular rates (i.e. $\omega_z \neq constant$), the observable terms are

$$\delta v_y,\ \delta v_z,$$
$$b_{ay} - g\phi_x - \omega_z\delta v_x,\ b_{az} - f_y\phi_x,$$
$$gb_{gx} - \omega_z(b_{ax} + g\phi_y) - \omega_z g\phi_y - \omega_z^2(\delta v_y + v_x\phi_z) - \dot{\omega}_z\delta v_x,$$
$$\omega_z v_x(b_{gx} - \omega_z\phi_y) - \dot{\omega}_z v_x\phi_x,$$
$$2\omega_z g(b_{gy} + \omega_z\phi_x) - \omega_z^2(b_{ay} - g\phi_x - v_x b_{gz} - \omega_z\delta v_x) + \ldots$$
$$-\dot{\omega}_z[2b_{ax} + 3g\phi_y + 3\omega_z(\delta v_y + v_x\phi_z)],$$
$$\omega_z^2 v_x(b_{gy} + \omega_z\phi_x) + \dot{\omega}_z v_x(b_{gx} - \omega_z\phi_y) - \ddot{\omega}_z v_x\phi_x,$$
$$\ldots$$

As the vertical angular rate changes, it is possible to distinguished the coupled item $\delta v_y + v_x\phi_z$ from its whole coupling term, which means this coupled item can be estimated. Also, the pitch estimation can be enhanced because of the change of the vertical angular rate.

It is illustrated by the analysis above that

1. The contributions of NHC on a certain state depend on both the current vehicle dynamic and the relative error magnitude of the coupled state under the current vehicle dynamic. The state with a distinctly larger error can be corrected with priority.

2. The NHC can enhance the estimation of the yaw and pitch while the vehicle is turning or changing its forward speed. This property is especially important when the land vehicle is moving in the tunnel without the aid of GNSS. Any accelerating or turning motions will enhance the NHC contributions to the estimation of both the yaw and the pitch. However, the contribution to the yaw will be weak with a low vehicle speed or zero speed.
3. The NHC has significant effects in controlling the roll, especially under low vehicle dynamics (e.g. static or uniform linear motions).
4. The gyro biases along cross-track and vertical directions (y and z axis) of the vehicle can be estimated when the vehicle is accelerating or turning. By contrast, the forward gyro bias has strong observability under low vehicle dynamics but may be disturbed while accelerating or turning.
5. The unobservable part of the accelerometer biases along the cross-track direction always couples with that of the roll; the forward accelerometer bias can only be estimated when the vehicle turns, and it couples with the pitch then; the vertical accelerometer bias has relatively better observability, especially under low vehicle dynamics.

In order to verify the theoretical analysis, corresponding simulations and covariance analysis were carried out.

4. Simulation Analysis Method

The whole simulation analyzing process included two parts: simulation and error analysis. The simulation process was done by the GNSS/INS simulation software developed by the GNSS Research Center at Wuhan University. The block diagram of simulation is shown in Fig. 2. The simulation was mainly comprised of trajectory generation and the modeling of both the IMU and GNSS errors. The NHC was used as the measurement update, instead of the GNSS measurements. The error analysis includes the navigation computation and the covariance analysis.

5. Simulation Tests and Results

5.1 Simulation tests

A series of maneuvers were simulated to cover the typical dynamics mentioned above. The trajectories are described in Table 1. The error characteristics of simulated IMU (typical MEMS grade) are shown in Table 2. The simulated motion information is shown in Fig. 3. The four subplots denote horizontal trajectory, velocities (in the n-frame), specific forces and angular rates (both in the b-frame), respectively.

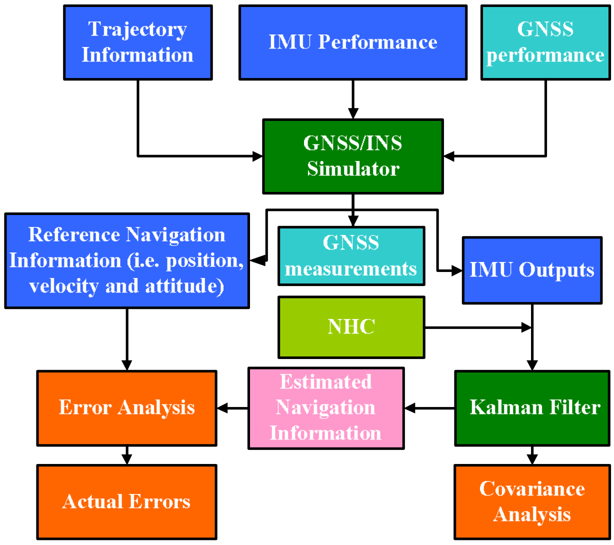

Fig. 2. Block diagram of the simulation and error analysis

Table 1. Simulated vehicle motions

Time segment (Sec)	Motion description
0-60	Static
60-70	Forward speed increases linearly in time (acceleration = 2 m/s^2)
70-130	Uniform linear motion (speed = 20 m/s)
130-140	Forward speed decreases linearly in time (acceleration = -1 m/s^2)
140-150	Turn +90 degrees with constant angular acceleration
150-180	Uniform linear motion (speed = 10 m/s)
180-240	Motion with sinusoidal varying forward acceleration (forward acceleration changes with Amplitude = 4m/s^2 and Period = 20s)
240-250	Uniform linear motion (speed = 10 m/s)
250-310	The first 3 second angular rate rises from 0 to 18 deg/s; then uniform angular motion (angular rate =18 deg/s); the last 3 second angular rate falls to 0 deg/s
310-320	Uniform linear motion (speed = 10 m/s)
320-380	Motion with sinusoidal varying angular rates (speed = 10 m/s; angular rate changes with Amplitude = 36deg/s and Period = 20s)

Table 2. Simulated IMU characteristics

	Simulated errors	Values of errors
IMU	Gyro bias instability	Modeled as first-order Gauss-Markov process σ=36 °/hr, τ=100 sec
	Gyro white noise (ARW)	0.3 deg/sqrt(hr)
	Accel. bias instability	Modeled as first-order Gauss-Markov process σ=1000 mGal, τ=100 sec
	Accel. white noise (VRW)	0.12 m/s/sqrt(hr)
	Data rate	100 Hz

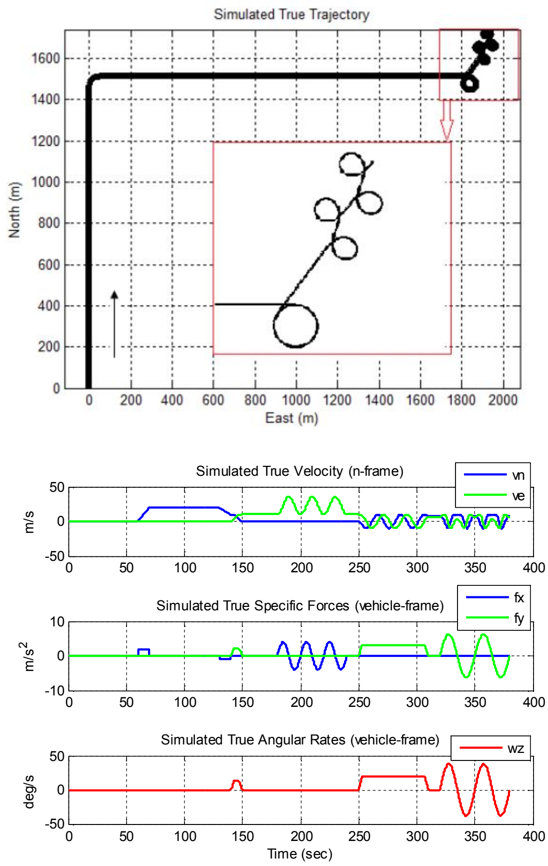

Fig. 3. Simulated trajectory, velocities, specific forces and angular rates

5.2 Simulation results and analysis

The IMU data simulated above were processed by the navigation algorithm described in Section 2.1, with only the NHC updates. The standard deviations of the velocity measurements from the NHC were set as 0.1 m/s. Here the simulation results analysis will focus on the attitude estimation and the sensor errors estimation, since they are the key states that have an impact the navigation performance. The curves of the attitude errors (i.e. the standard deviations of estimated errors given by Kalman filter) and the sensor biases errors are shown in Fig. 4 and Fig. 5, respectively.

According to Fig.4 and Fig.5, during the whole navigation process (380 sec), the NHC constrains all the attitude errors: the yaw error under 6 deg, the pitch error under 1 deg and the roll error under 0.5 deg, which is remarkable for a typical low-cost MEMS IMU without GNSS updates. Specifically, the NHC has different contributions to the estimation of both attitudes and sensor biases under different dynamics, as shown by the theoretical analysis above. Details of these phenomena are commented as follows.

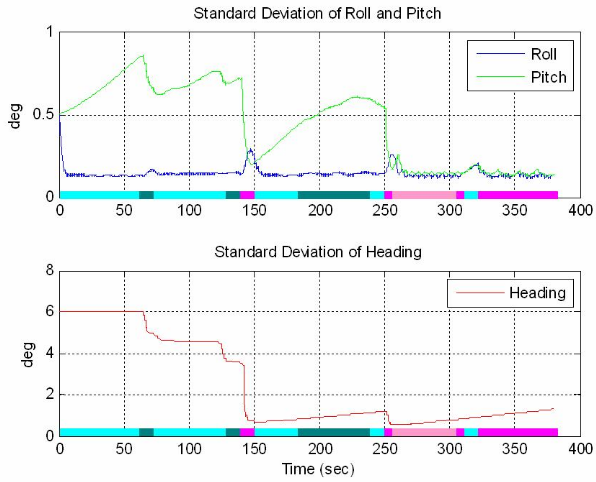

Fig. 4. Standard deviations of attitude estimation errors
(Cyan: static / Uniform linear motion;
dark green: variable acceleration;
pink: uniform circular motion;
magenta: variable angular velocity)

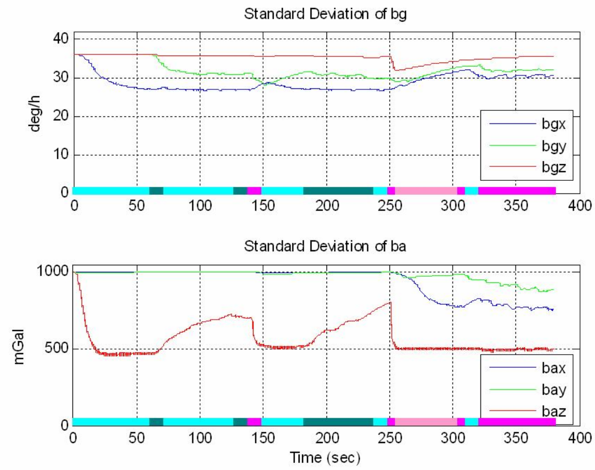

Fig. 5. Standard deviations of sensor bias estimation errors
(Cyan: static / Uniform linear motion;
dark green: variable acceleration;
pink: uniform circular motion;
magenta: variable angular velocity)

When the vehicle was stationary in 0-60s, x-axis gyro bias converged rapidly as well as z-axis accelerometer bias. The estimate of the roll was also enhanced to the similar magnitude as the y-axis accelerometer bias error divided by the gravity value. However, neither the pitch nor the yaw was enhanced because of their poor observability.

The vehicle began to move and speed up during 60-70 s, then both the yaw and the pitch were estimated, while the roll was disturbed slightly.

During 140-150 s when the vehicle was turning, both the yaw and the pitch converged sharply, while the roll was disturbed. At the same time, the estimation of the y-axis gyro bias was enhanced while that of the x-axis gyro bias was weakened.

During 180-240s when the vehicle experienced varying accelerations, both the pitch and the yaw still showed a diverging trend, which was not expected. This might be because that the pitch and yaw errors had already dropped to a relatively low level (compared with its coupled states) in the previous dynamics. They looked diverging, but this is actually because they were on their way to the new balance points, which were larger than their current level.

The yaw diverged in the last 120 s (i.e. 260-380s, the vehicle made uniform angular motions or motions with sinusoidal varying angular rates), which was also not expected. This might be due to the fact that although the vehicle made turnings during the last 120s, the moving range of the vehicle was limited (as shown in Fig 3) so that the vehicle was experiencing rather low dynamics (as if no much motion within a small area).

Generally speaking, the simulation results in this section match the observability analysis of the NHC/INS integration for land vehicle applications.

6. Conclusion

The contributions of the non-holonomic constraints (NHC) for inertial navigation have been studied from the perspective of observability. The effects of the NHC to each navigation states under different vehicle dynamics have been analyzed separately. Both the theoretical analysis and the simulation tests have shown that the contributions of the NHC to certain navigation state depend on both the current vehicle dynamic and the relative error magnitude of the coupled states under the current vehicle dynamic; both the accelerating and turning motions can enhance the contributions of the NHC to the estimation of both the yaw and the pitch, and this contribution will be stronger with a higher vehicle speed; the NHC has significant effects on controlling the roll in all motion conditions. Furthermore, the effects of the NHC on the estimation of the IMU sensor errors have also been analyzed in detail by the observability analysis. The outcomes of this paper have shown that the proposed observability analysis can be beneficial to the full utilization of the NHC or other priori information for inertial navigation.

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