*Journal of Global Positioning Systems (2010)*
*Vol.9, No.2 :156-165*
*DOI: 10.5081/jgps.9.2.156*

Journal of Global Positioning Systems
2010
© CPGPS

# A Modified Inverse Integer Cholesky Decorrelation Method and Performance on Ambiguity Resolution

**Jun Wang, Yanming Feng, Charles Wang**
*Queensland University of Technology, Brisbane, Australia*

**Abstract**

One of the research focuses in dealing with integer least squares problem is the decorrelation technique to improve the efficiency of the integer parameter search progress. It remains a challenging issue and becomes even more critical in processing multi-GNSS signals. Currently, there are three main decorrelation techniques being employed: the integer Gaussian decorrelation, the Lenstra–Lenstra–Lovász (LLL) algorithm and the inverse integer Cholesky decorrelation (IICD) method. To measure the performance of decorrelation techniques, the condition number is usually used as the criterion. Additionally, the number of grid points in the search space can be directly utilized as a performance measure according to the decorrelation purpose. The success rate of integer bootstrapping is also calculated in terms of studying the ambiguity resolution reliability.

This paper presents a modified inverse integer Cholesky decorrelation (MIICD) method to improve the decorrelation performance out the other three techniques. Decorrelation performance is evaluated based on the condition number of the decorrelation matrix and the number of search candidates. Performance parameters are compared using both simulation and real data. The simulation experiment scenarios employ the isotropic probabilistic model using a predefined eigenvalue and without any geometry or weighting system constraints. Simulation analysis shows that MIICD method outperforms other three methods in terms of condition numbers achieved. The real data experiment scenarios involve both single and dual constellations cases. Experimental results demonstrate that in the single constellation case, the condition number of MIICD is smaller than that of LAMBDA over 78.65% times while the number of search candidate points is smaller over 98.92% of time. In the dual constellation case, these two numbers are 98.78% and 100% respectively

**Keywords**: Modified Inverse Integer Cholesky Decorrelation, LLL, Condition Numbers, Ambiguity Resolution

## 1. Introduction

Integer ambiguity resolution is the key to high precision positioning using carrier phase measurements from Global Navigation Satellite System (GNSS). Given the GNSS linear observation equations

$$\mathbf{L} = \mathbf{A}\boldsymbol{\delta}\mathbf{x} + \mathbf{B}\mathbf{N} + \mathbf{e} \tag{1}$$

and the criterion:

$$\min_{\boldsymbol{\delta}\mathbf{x},\mathbf{N}} \left\{ \left\| \mathbf{L} - \mathbf{A}\boldsymbol{\delta}\mathbf{x} - \mathbf{B}\mathbf{N} \right\|^2_{\mathbf{Q}_L^{-1}}, \boldsymbol{\delta}\mathbf{x} \in R^n, \mathbf{N} \in Z^p \right\} \tag{2}$$

where $\mathbf{L}$ is a vector of 'observed minus computed' double-difference (DD) observations; $\mathbf{A}$ is the design matrix for the vector of real-valued unknowns $\boldsymbol{\delta}\mathbf{x}$ ; $\mathbf{B}$ is the design matrix for the vector of integer DD ambiguities $\mathbf{N}$; $\mathbf{Q}_\mathbf{L}$ is the corresponding variance matrix of observables and $\mathbf{e}$ is the vector of unmodelled error and measurement noise.

Solving the above mixed integer least-squares (MILS) problem has proved to be equivalent to the solution of the integer least-squares (ILS) problem:

$$\min_{\mathbf{N}} \left\{ \left\| \hat{\mathbf{N}} - \mathbf{N} \right\|^2_{\mathbf{Q}_{\hat{\mathbf{N}}}^{-1}}, \mathbf{N} \in Z^p \right\} \tag{3}$$

where $\hat{\mathbf{N}}$ is a float ambiguity vector, with the corresponding variance-covariance matrix $\mathbf{Q}_{\hat{\mathbf{N}}}$ . For more details on the procedure of solving MILS or ILS, see (Teunissen, 1995; Hassibi & Boyd, 1998; Grafarend, 2000; Chang & Zhou, 2007).

The integer ambiguity search space is defined as

$$(\hat{\mathbf{N}} - \mathbf{N})^T \mathbf{Q}_{\hat{\mathbf{N}}}^{-1} (\hat{\mathbf{N}} - \mathbf{N}) \le \chi^2 \tag{4}$$

It is actually a hyper-ellipsoid centered at $\hat{\mathbf{N}}$, its shape and orientation are governed by $\mathbf{Q}_{\hat{\mathbf{N}}}$ and its size can be controlled by $\chi^2$. In general, $\mathbf{Q}_{\hat{\mathbf{N}}}$ has high correlation since the DD operation and correlation between measurements errors. Hence, the integer ambiguity search space is highly elongated. In order to make the search process more efficient, different decorrelation techniques have been developed. The essence of decorrelation is to apply an admissible integer unimodular matrix $\mathbf{Z}$ to eliminate the off-diagonal elements of $\mathbf{Q}_{\hat{\mathbf{N}}}$ or reduce the size of the correlation coefficients. This can be expressed as

$$\hat{\mathbf{N}}_{\mathbf{dec}} = \mathbf{Z}\hat{\mathbf{N}},\ \mathbf{N}_{\mathbf{dec}} = \mathbf{Z}\mathbf{N},\ \mathbf{Q}_{\hat{\mathbf{N}}_{\mathbf{dec}}} = \mathbf{Z}\mathbf{Q}_{\hat{\mathbf{N}}}\mathbf{Z}^{\mathbf{T}} \qquad (5)$$

Therefore the search space (4) can be transformed as

$$(\hat{\mathbf{N}}_{\mathbf{dec}} - \mathbf{N}_{\mathbf{dec}})^T \mathbf{Q}_{\hat{\mathbf{N}}_{\mathbf{dec}}}^{-1} (\hat{\mathbf{N}}_{\mathbf{dec}} - \mathbf{N}_{\mathbf{dec}}) \le \chi^2 \qquad (6)$$

The condition number is usually used to indicate the performance of decorrelation methods. For instance, the well-known least-squares ambiguity decorrelation adjustment (LAMBDA) method is based on integer Gaussian decorrelation (Teunissen et al., 1995, 1996 and 1998). A detailed description and implementation of this method is referred to De Jonge & Tiberius, (1996). Another algorithm named Lenstra–Lenstra–Lovász (LLL) was originally developed for lattice basis reduction, which can also be used to reduce the condition number of matrix (Lenstra, Lenstra, & Lovász, 1982). This algorithm was suggested for the decorrelation of the integer ambiguities by Hassibi & Boyd (1998) and Grafarend (2000). Based on a modified LLL algorithm, Chang and Zhou (2007) developed a Matlab package for solving MILES problems and demonstrated higher computation efficiency than LAMBDA. Xu (2001) developed a random simulation approach to compare the performance of different decorrelation method, but the simulation is more general, without referring to any particular satellite-receiver geometry, observation span and measurement weightings. This non-informativeness guarantees the statistical fairness of comparing different methods numerically because these three factors may favor a particular method. Xu also proposed an inverse integer Cholesky decorrelation method and demonstrated that this method outperformed LAMBDA and LLL method. However, the performance of these decorrelation methods in dealing with practical high dimension cases remains unknown (Xu, 2001; Svendsen, 2006). In the near future, more frequency signals, e.g. L1, L2 and L5 and more navigation satellites systems, e.g. GPS and Galileo could be used. Introducing more observations from three frequency signals and dual constellations changes the condition number of $\mathbf{Q}_{\hat{\mathbf{N}}}$. Using the same data set as described in Section 4, Figure 1 plots the condition number of $\mathbf{Q}_{\hat{\mathbf{N}}}$ and the corresponding decorrelated matrix $\mathbf{Q}_{\hat{\mathbf{N}}\mathbf{dec}}$ for both double and triple frequencies cases. It is clearly observed that the $\mathbf{Q}_{\hat{\mathbf{N}}\mathbf{dec}}$ condition numbers of triple frequencies are larger than that of double frequencies. Figure 2 compares the condition numbers of $\mathbf{Q}_{\hat{\mathbf{N}}}$ and $\mathbf{Q}_{\hat{\mathbf{N}}\mathbf{dec}}$ between single GPS constellation and the simulated dual constellations, which refer to the combination of GPS measurements data sets recorded at two epochs separated by a few hours for data analysis (Feng, 2005) as outlined in Section 3.2 It is seen that the condition numbers of $\mathbf{Q}_{\hat{\mathbf{N}}\mathbf{dec}}$ are larger than these for the single constellation. In this research effort, a modified inverse integer Cholesky decorrelation method is proposed to further decorrelate $\mathbf{Q}_{\hat{\mathbf{N}}}$ and reduce the conditional numbers.

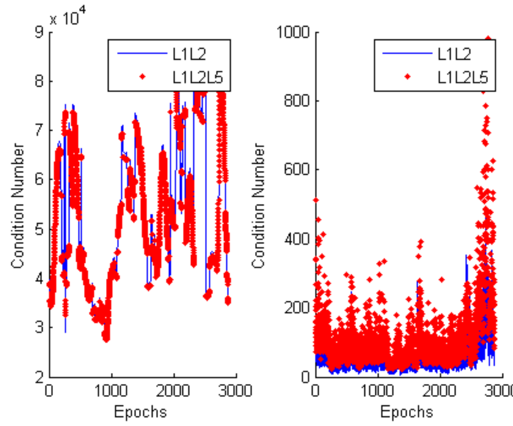


Figure 1: Condition numbers of $\mathbf{Q}_{\hat{\mathbf{N}}}$ and $\mathbf{Q}_{\hat{\mathbf{N}}\mathbf{dec}}$ in L1L2 and L1L2L5 cases. Left plot: the float ambiguity variance-covariance matrix $\mathbf{Q}_{\hat{\mathbf{N}}}$ Right plot: the decorrelated ambiguity variance-covariance matrix $\mathbf{Q}_{\hat{\mathbf{N}}\mathbf{dec}}$

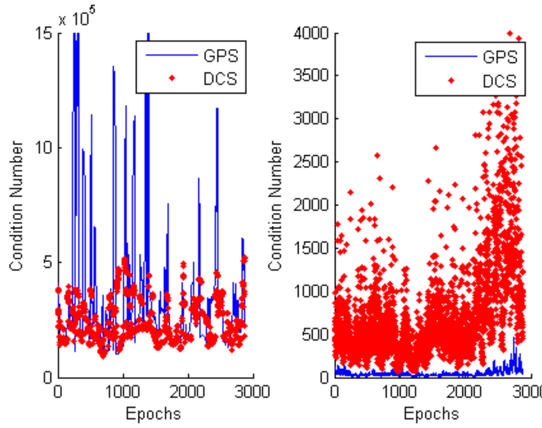


Figure 2: Condition numbers of $\mathbf{Q}_{\hat{\mathbf{N}}}$ and $\mathbf{Q}_{\hat{\mathbf{N}}\mathbf{dec}}$ in GPS and dual donstellations cases. Left plot: the float ambiguity variance-covariance matrix $\mathbf{Q}_{\hat{\mathbf{N}}}$ Right plot: the decorrelated ambiguity variance-covariance matrix $\mathbf{Q}_{\hat{\mathbf{N}}\mathbf{dec}}$

On the other hand, the condition properties of decorrelation methods may not necessarily link to the ambiguity searching efficiency. A simple way is to count the grid points within the search space. Although the volume of a search space has been demonstrated to be a fair approximation of the number of grid points on the average (Teunissen et al., 1996), the relation between the grid point number and condition number have not been specifically examined. In the later context of this work, we will also compare the number of search grid points of different decorrelation methods to show their dependence.

Since the success rate of bootstrapping integer solutions could be a very good approximation of ILS success rate (Teunissen, 1998; Verhagen, 2003) have numerically demonstrated close agreement with actual statistical results by Feng and Wang (2011), in this contribution, we will compute the success rate of ambiguity bootstrapping integer estimation with different decorrelation methods. It is also noticed that the work by Henkel (2007 and 2009) investigated the impact of biases inflation by integer decorrelation transformation.

The rest of paper is organized as follows. Section 2 briefly introduces different decorrelation techniques and introduces the modified inverse integer Cholesky decorrelation method. Section 3 presents the random simulation strategies, the concept of generating dual constellations and the criterion used to compare the performance of different decorrelation methods. Section 4 discusses the experimental results from four computation scenarios. The main findings of the paper are summarized in the final section.

## 2. Decorrelation techniques

The variance-covariance matrix $\mathbf{Q}_{\hat{\mathbf{N}}}$ of DD float ambiguities possesses highly-correlated off-diagonal elements. The goal of a decorrelation process is to find a unimodular matrix Z to reduce the off-diagonal elements or reduce the size of the correlation coefficients. Since the matrix Z should be admissible and integer, the absolute decorrelation is impossible in most cases. The LAMBDA method based on integer Gaussian decorrelation has been proved to be highly efficient for ambiguity resolution in most situations (Teunissen et al., 1995 and 1997). Although the LLL algorithm is developed for lattice basis reduction, the method can also be used to reduce the correlation of $\mathbf{Q}_{\hat{\mathbf{N}}}$ in ILS (Sanzheng, 2008). The inverse integer Cholesky decorrelation (IICD) method uses the Cholesky decomposition instead of Gaussian decomposition for GPS decorrelation (Xu, 2001). Based on IICD method, a modified inverse integer Cholesky decorrelation (MIICD) is proposed.

### 2.1 Integer Gaussian decorrelation

Integer Gaussian decorrelation is actually performed as a sequence of integer Gaussian eliminations and permutations. Assuming there exist three elements $q_{ii}$, $q_{jj}$ and $q_{ij}$ of $\mathbf{Q}_{\hat{\mathbf{N}}}$ that satisfy $\left|q_{ij}\right|/\min(q_{ii},q_{jj})>1/2$, then the unimodular matrix can be constructed as

$$\mathbf{Z_1}=\begin{bmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & \vdots & \ddots & & \\ & & -\left[q_{ij}/q_{ii}\right] & \cdots & 1 & \\ & & & & \ddots & \\ & & & & & 1 \end{bmatrix} \tag{7}$$

if $q_{ii}\le q_{jj}$ ,or

$$\mathbf{Z_1}=\begin{bmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & \cdots & -\left[q_{ij}/q_{\mathrm{jj}}\right] & \\ & & & \ddots & \vdots & \\ & & & & 1 & \\ & & & & & \ddots & \\ & & & & & & 1 \end{bmatrix} \tag{8}$$

if $q_{ii}>q_{jj}$. Here the operator [ ] denotes rounding to the nearest integer. Repeating the above steps, the final **Z** transformation matrix can be expressed as

$$\mathbf{Z}=\mathbf{Zn}\cdots\mathbf{Z_2Z_1} \tag{9}$$

Thus, the decorrelated matrix $\mathbf{Q}_{\hat{\mathbf{N}}\mathbf{dec}}$ can be obtained by equation (5).

### 2.2 Lenstra–Lenstra–Lovász algorithm

The LLL algorithm was first introduced for decorrelation in GPS ambiguity resolution by Hassibi and Boyd (1998), followed with the contributions by Grafarend (2000), Xu (2001) and Chang (2007). The original LLL algorithm uses the integer Gram-Schmidt orthogonalization process, however the Givens reflection based LLL algorithm can be numerically more robust performed in floating-point arithmetic (Luk & Qiao, 2007; Luk & Tracy, 2008). Since the matrix $\mathbf{Q}_{\hat{\mathbf{N}}}$ is positive and definite, it can always be factorized as $\mathbf{Q}_{\hat{\mathbf{N}}}=\mathbf{V}^{\mathbf{T}}\mathbf{V}$. Compute the reduced or almost orthogonal basis $\mathbf{V_0}$, so that $\mathbf{V_0}=\mathbf{V}\mathbf{Z}^{-1}$, where **Z** is unimodular. After these steps, we have

$$\mathbf{Q}_{\hat{\mathrm{N}}} = \mathbf{V}^T\mathbf{V} = (\mathbf{V}_0\mathbf{Z})^T\mathbf{V}\mathbf{Z} = \mathbf{Z}^T\mathbf{V}_0^T\mathbf{V}_0\mathbf{Z} \qquad (10)$$

Due to V0 is almost orthogonal, the target of decorrelation can be achieved with $\mathbf{Q}_{\hat{\mathrm{N}}\mathbf{dec}} = \mathbf{V}_0^T\mathbf{V}_0$ .

### 2.3 Inverse integer Cholesky decorrelation (IICD) method

The inverse integer Cholesky decorrelation (IICD) method applies the $\mathbf{LDL}^T$ factorization as follows:

$$\mathbf{Q}_{\hat{\mathrm{N}}} = \mathbf{LDL}^T \qquad (11)$$

where **L** is unit lower triangular matrix, and **D** is a diagonal matrix with positive elements.

Although **L** cannot be directly used for ambiguity decorrelation due to the real-valued elements, [**L**] is obviously unimoudular as well as [**L**$^{-1}$]. Thus, we $\mathbf{Z}_1 = \left[\mathbf{L}^{-1}\right]$ and can compute the decorrelated matrix as

$$\mathbf{H}_1 = \mathbf{Z}_1\mathbf{Q}_{\hat{\mathrm{N}}}\mathbf{Z}_1^T \qquad (12)$$

Since in most cases $\mathbf{Z}_1$ is not equivalent to **L**, $\mathbf{H}_1$ is no longer diagonal. Repeating the process like

$$\mathbf{H}_n = \mathbf{Z}_n\mathbf{H}_{n-1}\mathbf{Z}_n^T \qquad (13)$$

until the condition number of Hn reach the predetermine value, the final decorrelation can be express as (Xu, 2001)

$$\mathbf{Q}_{\hat{\mathrm{N}}\mathbf{dec}} = (\mathbf{Z}_n\cdots\mathbf{Z}_2\mathbf{Z}_1)^T\mathbf{Q}_{\hat{\mathrm{N}}}(\mathbf{Z}_n\cdots\mathbf{Z}_2\mathbf{Z}_1) \qquad (14)$$

To obtain larger off-diagonal elements of **L**, we may rearrange the diagonal elements of $\mathbf{Q}_{\hat{\mathrm{N}}}$ and $\mathbf{H}_i$ in ascending order. Before finishing this section, we would like to make some arguments on this method. Firstly, what should be the predetermined condition number of the decorrelated matrix? The answer is not easy to say, because it relates to the dimension and formation of the original matrix. Secondly, since this method involves iteration process, sorting and stopping criteria would be very important for the IICD method (private communication with Dr. Xu on 25 June 2010). Simply comparing the condition number of $\mathbf{H}_\mathbf{n}$ and $\mathbf{H}_\mathbf{n-1}$ can lead to wrong decision, because it is likely to happen that the condition numbers of $\mathbf{H}_\mathbf{n}$ and $\mathbf{H}_\mathbf{n-1}$ are not in the strictly descending order. To overcome this shortcoming of IICD, we will propose a new method in the next section.

### 2.4 Modified inverse integer Cholesky decorrelation (MIICD) method

Instead of using the predetermined condition number as the iteration stopping criteria, we consider applying whether the abs($\mathbf{Z}_\mathrm{n}$) is an identity matrix to stop the process of inverse integer Cholesky decorrelation, where abs( ) is the absolute value operator. In addition, we may also rearrange the diagonal elements of **H** in descending order after stopping iteration and repeat the decorrelation process. It is observed that the condition numbers of $\mathbf{H}_\mathbf{i}$ usually decrease with fluctuation, so we will record the condition number of $\mathbf{H}_\mathbf{i}$ and transformation matrix $\mathbf{Z}_\mathbf{i}$ each time while conducting the procedure of decorrelation. This function allows us to be able to find the smallest condition number by searching Hi. Another iteration stopping criteria in this method is the predetermined iteration number. Figure 3 depicts this modified inverse integer Cholesky decorrelation method.

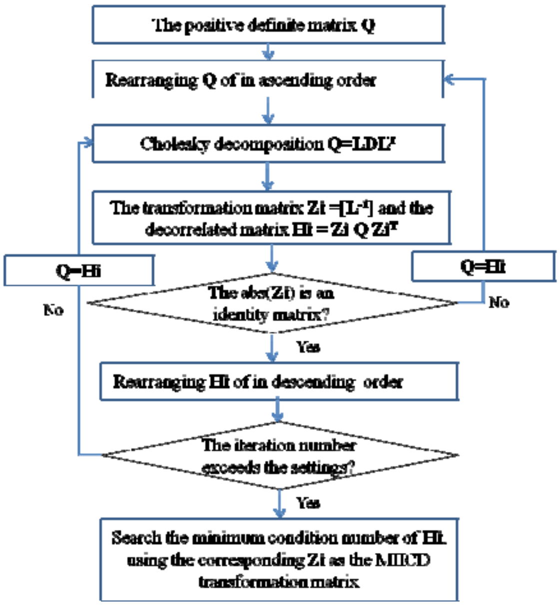


Figure 3: Flowchart if the modified inverse integer Cholesky decorrelation method.

## 3. Random simulation and measuring performance

In order to study the numerical performance of different decorrelation methods, an isotropic probabilistic model is used to simulate a positive definite matrix instead of a particular one (Xu, 2001 and 2002). In addition we apply the concept of Virtual Galileo Constellation (VGC) to generate useful data sets of dual-constellations (Feng, 2005). The condition number is usually used to be an index of decorrelation methods performance (Svendsen, 2006), but it might not directly reflect the integer candidates search efficiency. Therefore, the integer candidates search numbers can be compared with different decorrelation method.

### 3.1 Random simulation method

Any positive definite matrix **Q** can be decomposed by singular value decomposition as

$$\mathbf{Q} = \mathbf{U}\Lambda\mathbf{U}^T \tag{15}$$

where **U** is the normalized orthogonal eigenvector matrix and $\Lambda$ is the diagonal matrix with positive eigenvalues $\lambda_1 \le \lambda_2 \le \cdots \le \lambda_n$. Then the simulation of **Q** is turned into design of **U** and $\Lambda$.

The isotropic probabilistic model is used to generation of an arbitrary **U**, which can be uniquely represented as

$$\mathbf{U}=\mathbf{U}_{\text{n(n-1)}} \cdots \mathbf{U}_{32}\mathbf{U}_{\text{n1}} \cdots \mathbf{U}_{31}\mathbf{U}_{21} \tag{16}$$

where

$$\mathbf{U_{ij}} = \begin{bmatrix} \mathbf{I}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \cos\theta_{ij} & \mathbf{0} & \sin\theta_{ij} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\sin\theta_{ij} & \mathbf{0} & \cos\theta_{ij} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_3 \end{bmatrix}, \tag{17}$$

$\mathbf{I}_1$, $\mathbf{I}_2$ and $\mathbf{I}_3$ are the identity matrices of suitable orders, **0** is a zero matrix or vector and $-\pi/2 \le \theta_{ij} \le \pi/2$ (Xu, 2001and 2002).

The next step is to design $\Lambda$ which is related to the eigenvalues of **Q**. Since the condition number of **Q** can be expressed as follows

$$cond(\mathbf{Q}) = \lambda_{\max} / \lambda_{\min} \tag{18}$$

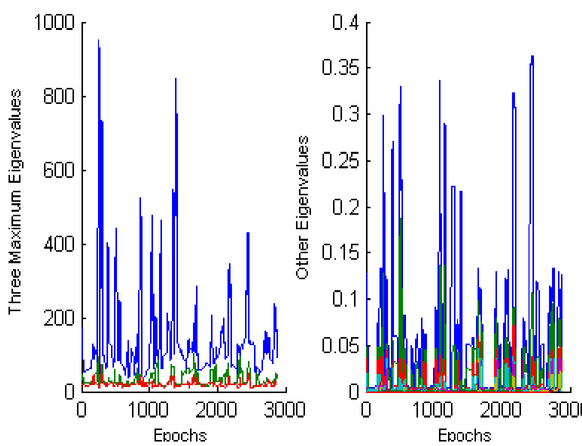


Figure 4: The eigenvalues partition of the covariance matrix of the float ambiguities. Left plot: the three largest eigenvalues; Right plot: the remaining eigenvalues

Although we can generate all the eigenvalues equally separated with predetermined condition number or ratio, more complex assumption or constrains can be imposed. For single baseline geometry-based model, there are only three independent DD ambiguities and other DD ambiguities can be derived from those(Li & Shen, 2010). Figure 4 shows the eigenvalue partition of the covariance matrix of the float ambiguities in single baseline. Obviously there are three large ones and the remaining eigenvalues are significantly small.

### 3.2 Virtual Galileo Constellation (VGC) Model

The concept of VGC is to combine the GPS measurements data sets recorded at two epochs separated by a few hours to form dual constellations for data analysis. Feng (2005) showed that the separation can range from 1 to 2 hours. For GPS and VGC data sets, one can obtain the linear equations based on (1)

$$\begin{bmatrix} \mathbf{L}_{\text{gps}} \\ \mathbf{L}_{\text{gal}} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{\text{gps}} & \mathbf{B}_{\text{gps}} & \mathbf{0} \\ \mathbf{A}_{\text{gal}} & \mathbf{0} & \mathbf{B}_{\text{gal}} \end{bmatrix} \begin{bmatrix} \delta\mathbf{X} \\ \mathbf{N}_{gps} \\ \mathbf{N}_{gal} \end{bmatrix} + \begin{bmatrix} \mathbf{e}_{gps} \\ \mathbf{e}_{gal} \end{bmatrix} \tag{19}$$

where the subscript "gal" represents Galileo. It is noted that in (19), two data sets are assumed to have the same coordinates systems, but different sets of ambiguity parameters.

### 3.3 Measuring performance

Condition numbers are often used to compare the performance of different decorrelation techniques, which only reveal the ratio of the square of semi-major axis and semi-minor axis of search ellipsoid. It can only partially reflect the ILS search progress efficiency; thus, the search numbers of grid points are also used to compare the impact of different decorrelation methods on search efficiency within the same search method. The details on how to compute the search numbers of candidates are referred to the instruction of LAMBDA and MILES (De Jonge & Tiberius, 1996; Chang & Zhou, 2007).

Furthermore, the success rate is computed considering ambiguity reliability requirements. The actual success rate of ILS is difficult to calculate; nevertheless the success rate of bootstrapping integer solution is a lower bound and a very good approximation of ILS (Teunissen, 1998; Feng & Wang, 2010), which can be computed as

$$\begin{aligned} &\mathrm{P}(\hat{\mathrm{N}}\,|_{ILS}) \ge \mathrm{P}(\hat{\mathrm{N}}\,|_{Boostrapping}) \\ &= \prod_{i=1}^{t} \int_{R_0} \frac{1}{(2\pi)^{1/2}\sigma_0\sqrt{\mathbf{Q_{i|I}}}} \exp[-\frac{(\mathbf{x}-\mathbf{z_i})^2}{2\sigma_0^2\mathbf{Q_{i|I}}}]\mathrm{dx} \end{aligned} \tag{20}$$

where the diagonal elements of $\mathbf{Q}_{i|I}$ can be calculated by factorization based on (11).

## 4. Experiments

The purpose of the experimental analysis is to examine the decorrelation performance of LAMBDA, LLL, IICD and MIICD methods in different situations. Four computation scenarios are set up as follows:

- Scenario 1: Performing LAMBDA, LLL, IICD and MIICD decorrelation with randomly simulated definite-positive covariance matrices;
- Scenario 2: Performing LAMBDA, LLL, IICD and MIICD decorrelation with randomly simulated definite-positive covariance matrices where eigenvalues are constrained to certain values as discussed in Section 3;
- Scenario 3: Performing LAMBDA and MIICD decorrelation in ILS processing of a real GPS data set for a 21 km baseline ;
- Scenario 4: Performing LAMBDA and MIICD decorrelation in ILS processing of the same data set as Scenario 3, but added with virtual GNSS data.

In Scenarios 1 and 2, 300 Q matrix samples are randomly generated for simulation experiments. We then set the condition number of original positive definite matrix Q based on the sample dimension size (as shown in Figure 5). The condition number is set as $1\times10^4$, if $4 \le \dim(\mathbf{Q}) \le 20$; or $1\times10^5$, if $20 < \dim(\mathbf{Q}) \le 24$, where dim () is the matrix dimension operator.

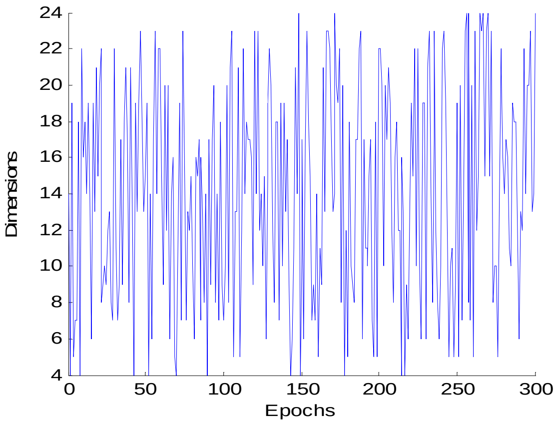


Figure 5: Dimensions of the 300 random simulation examples

Figure 6 shows the condition numbers of the original simulation matrix **Q** and the results for four decorrelation methods for Scenario 1. It is obvious that all decorrelation methods can significantly decrease the condition number of **Q**. Particularly, the condition numbers resulted from LLL and MIICD decorrelation are smaller than 200 and in most cases are smaller than 100. Meanwhile, the condition numbers resulted from other two methods, LAMBDA and IICD are slightly higher, mostly below 200 with occasional peaks to 400.

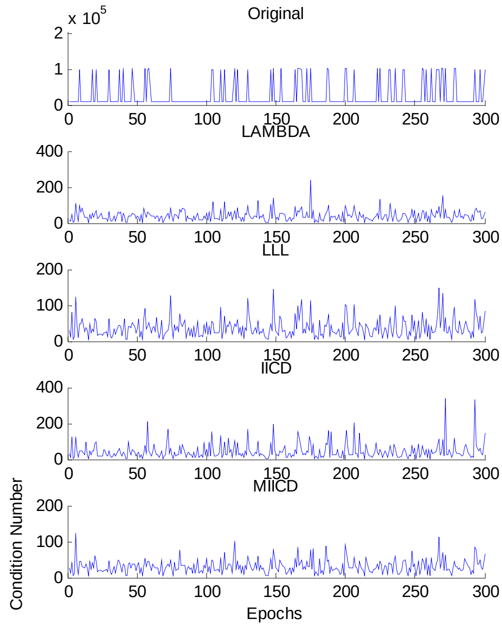


Figure 6: Condition numbers of simulated **Q** samples and results from LAMBDA, LLL, IICD and MIICD with Scenario 1

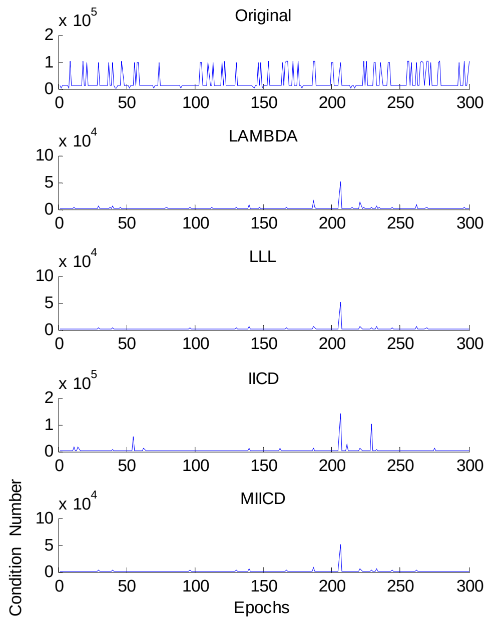


Figure 7: Condition Numbers of simulated **Q** samples, results from LAMBDA, LLL, IICD and MIICD in Scenario 2

For Scenario 2, constraint eigenvalues of **Q** were generated for performance evaluation as discussed in Section 3.1. The condition numbers of the original **Q** matrix and four decorrelation methods are shown in Figure 7. In this scenario, though most samples were successfully decorrelated by these methods, results indicate that the decorrelation may not occur at some epochs based on given stopping criteria on condition number comparison. For instance, at epoch 206, the decorrelation did not happen with IICD method.

Table 1 summarizes the events and percentages when the condition numbers of decorrelated matrices from different methods are smaller than those from AMBDA method. It is clear that MIICD has the best performance in terms of condition numbers in both of two scenarios. It shows that MIICD method outperformed other three methods in terms of the events with smaller condition numbers than LAMBDA method. In particular, without eigenvalue constraints in the decorrelated matrices, in 235 out of 300 samples, or at 78.33% of time, MIICD conditional numbers are lower than LAMBDA condition numbers. With eigenvalue constraints, the samples and percentages grow to 245 and 81.67% of times, respectively. Therefore, for simplicity, only MIICD and LAMBDA method are compared for scenarios 3 and 4.

For Scenario 3 and Scenario 4, a real GPS data set of 24 hours collected at sampling rate of 30 seconds is used. The virtual Galileo constellation (VGC) used in Scenario 4 is generated from the collected dataset with time latency of 2 hours.

Table 1: Lower condition number statistics derived from LLL, IICD and MIICD with respect to LAMBDA

| | Scenario 1 | Scenario 2 |
|---|---|---|
| LLL | 161 (53.67%) | 213 (71.00%) |
| IICD | 178 (59.33%) | 225 (75.00%) |
| MIICD | 235 (78.33%) | 245 (81.67%) |

The condition numbers of LAMBDA and MIICD methods for Scenario 3 and Scenario 4 have been computed and shown in Figure 8 and Figure 9 respectively. It can be clearly seen that the condition number results of these methods have similar trends and fluctuations in most cases except the MIICD has smaller condition numbers. In particular, the MIICD method has significant performance improvement in the dual constellation case where the peak condition number of LAMBDA is larger than 8000 while the peak MICCD condition number is about 1000.

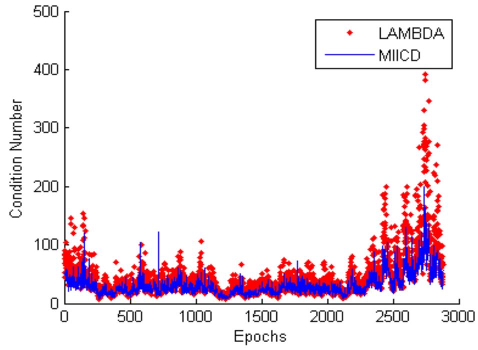


Figure 8: Condition numbers of Q matrices from a 24-h data set, resulting from LAMBDA and MIICD with Scenario 3.

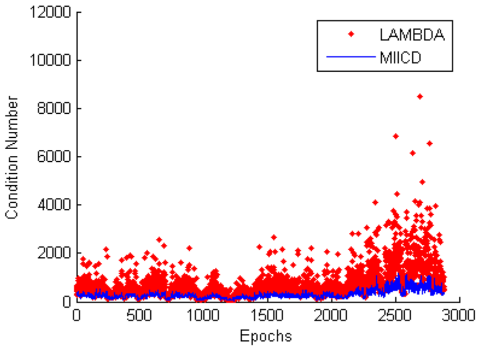


Figure 9: Condition numbers of Q matrices from a 24 h data set, resulting from LAMBDA and MIICD with Scenario 4

The search candidate numbers of LAMBDA and MIICD methods for Scenarios 3 and 4 are shown in Figure 10 and Figure 11 respectively. It can be clearly observed that the search candidate numbers of LAMBDA are generally larger with respect to these of the MIICD method. Similarly to the condition number results, the improvement in the search candidate numbers of MIICD method is more significant in the dual constellation case. For instance in Scenario 4, the search numbers of LAMBDA are around 1×105 between epochs 2400 and 2800, whereas the MIICD search numbers are mostly less than 200 with the peak of 4000 during the time.

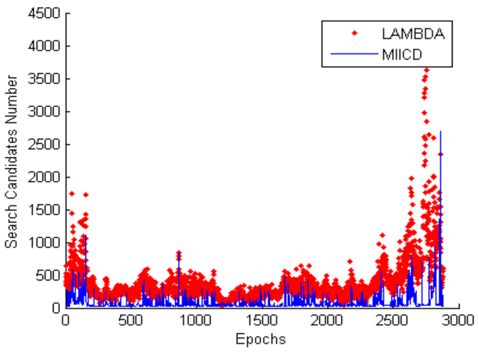


Figure 10: Search candidate numbers in the 24 h data set, resulting from LAMBDA and MIICD with Scenario 3

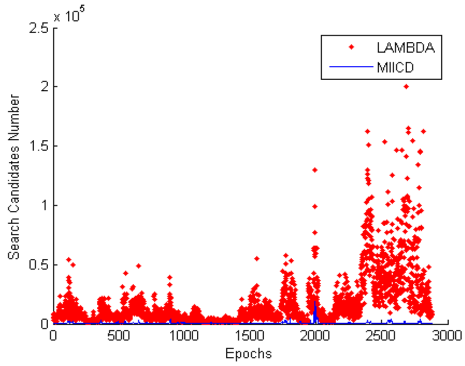


Figure 11: Search candidate numbers in the24 hours, resulting from LAMBDA and MIICD with Scenario 4

To investigate relations between condition numbers and search candidate numbers, we can either draw the scatter plots or calculate the correlation coefficients. Figure 12 shows the scatter plots of these two parameters. A linear dependence is clearly shown between the condition number and the search candidate number.

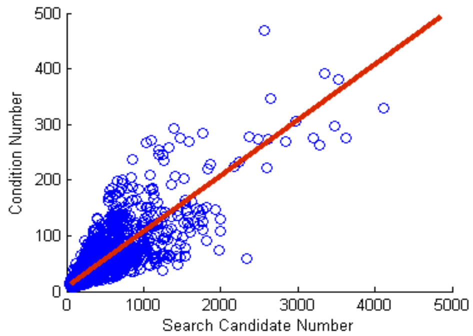


Figure 12: Scatter plots of the search candidate number against the condition number

Table 2 presents the correlation coefficients which also verify that the condition number is highly related to the candidate search number. On the other hand, the correlation coefficient 0.8050 also reveals that the condition numbers cannot totally be represented by the search candidate numbers.

Table 2: The correlation coefficients between search candidate numbers and condition numbers

| Correlation coefficient | Search candidate number |
|---|---|
| Condition numbers | 0.8050 |

The success rates of LAMBDA and MIICD method were computed with (20) for Scenario 3 Scenario 4 and shown in Figure 13 and Figure 14 respectively. It is seen that in most cases the computed success rates of LAMBDA are higher than MIICD, particularly for the dual constellation case as evidenced in Figure 15. However, the actual statistics for success rates are the same from both methods. Table 3 summarizes the statistics of LAMBDA and MIICD methods in the cases of Scenario 3 and Scenario 4.

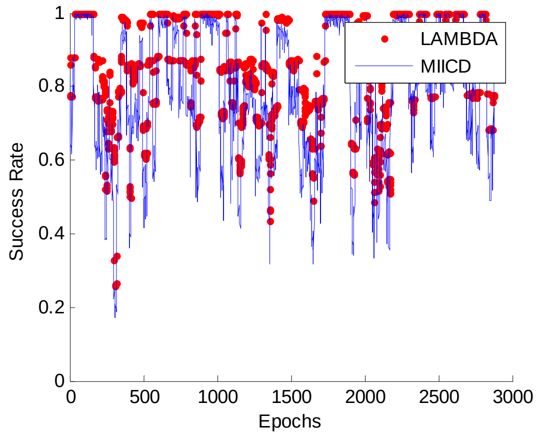


Figure 13: Computed success rates of the 24-h data set, resulting from LAMBDA and MIICD with Scenario 3

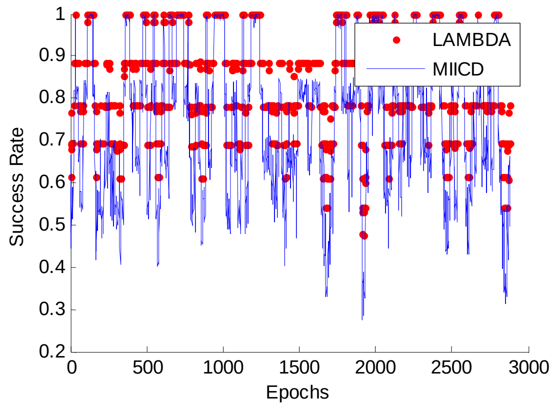


Figure 14: Computed success rates of the 24 h data set, resulting from LAMBDA and MIICD with Scenario 4

Table 3: MIICD with respect to LAMBDA: data epochs with Lower condition numbers and search numbers and success rates derived from the 24-h data set.

| | Scenario3 | Scenario 4 |
|---|---|---|
| Condition numbers | 2265 (78.65%) | 2816 (97.78%) |
| Search numbers | 2849 (98.92%) | 2880 (100%) |
| Computed success rates | 2878 (99.93%) | 2880 (100%) |
| Actual success rates | 0 (0%) | 0 (0%) |

From the above figures and tables, we can obtain the following useful observations:

- MIICD has better performance than LAMBDA in terms of condition numbers and search grid point numbers, especially in the high dimension case and the dual constellation case. For instance, 97.78% of the condition numbers and 100% of the search grid point numbers of MIICD method are smaller than those of LAMBDA method. In terms of the condition numbers, the improvement percentage (78.65%) of Scenario 3 is very close to the simulated case (78.33%) of Scenario 1;
- In terms of computed success rates of integer ambiguity bootstrapping solutions, the success rates of LAMBDA method is mostly higher than MIICD method. But both methods lead to the same actual success rates (100%). This may indicate that the bootstrapping success rate formula may not suit MIICD method well.

## 5. Conclusions

Effective decorrelation is the key to reliable and fast phase ambiguity resolution in GNSS real time data processing. Several decorrelation techniques have been developed and their performance have been discussed with a main focus on condition numbers. Although the inverse integer Cholesky decorrelation (IICD) method may outperform the LAMBDA method and LLL algorithm as shown through numerical analysis with random simulation ( Xu, 2001), its performance with real world data has not been reported.

In this contribution, we have proposed a modified inverse integer Cholesky decorrelation (MIICD). Four different experiments from respective simulation data and real data have demonstrated that further improvement has been achieved by MIICD. In general, results from both random simulation and real data have suggested that MIICD can provide superior performance in most of the situations. In particular, the MIICD method can significantly reduce the condition numbers, at 78.65% and 97.78% of times, search numbers at 98.92% and 100% of times in single and dual constellation cases, respectively, comparing with the LAMBDA method. This performance improvement demonstrates its potential benefits for real world GNSS ambiguity resolution data processing.

## Acknowledgments

This work was partially supported by CRC for Spatial Information project 1.04 "Delivering Precise Positioning Services in Regional Areas" 2007-2010. The first author acknowledges the financial support by China Scholarship Council (Grant No: 2008466001). Last but not least, the authors would like to thank Dr. Xu Peiliang from Kyoto University for his constructive suggestions and comments.

## Reference

Chang, X. W., & Zhou, T. (2007). ***MILES: MATLAB package for solving Mixed Integer LEast Squares problems.*** GPS Solutions, 11(4), 289-294.

De Jonge, P., & Tiberius, C. (1996). ***The LAMBDA method for integer ambiguity estimation: implementation aspects.*** Publications of the Delft Geodetic Computing Centre, 12.

Feng, Y. (2005). ***Future GNSS Performance Predictions Using GPS with a Virtual Galileo Constellation.*** GPS World, 16(3), 46-52.

Feng, Y. & Wang, J. (2010). ***Computed success rates of various carrier phase integer estimation solutions and their comparison with statistical success rates.*** Journal of Geodesy, 85: 93-103.

Grafarend, E. W. (2000). ***Mixed Integer-Real Valued Adjustment (IRA) Problems: GPS Initial Cycle Ambiguity Resolution by Means of the LLL Algorithm***. GPS Solutions, 4(2), 31-44.

Hassibi, A., & Boyd, S. (1998). ***Integer parameter estimation in linear models with applications to GPS***. IEEE Transactions on signal processing, 46(11), 2938-2952.

Henkel, P. A. G., Christoph (2007). ***Integrity Analysis of Cascade Integer Ambiguity Resolution with Decorrelation Transformations***. Paper presented at the National Technical Meeting of the Institute of Navigation (NTM '07), San Diego, California, USA.

Henkel, P., & Günther, C. (2010). ***Partial integer decorrelation: optimum trade-off between variance reduction and bias amplification***. Journal of Geodesy, 84: 51-63.

Lenstra, A. K., Lenstra, H. W., & Lovász, L. (1982). ***Factoring polynomials with rational coefficients***. Mathematische Annalen, 261(4), 515-534.

Li, B., & Shen, Y. (2010). ***Global Navigation Satellite System Ambiguity Resolution with Constraints from Normal Equations***. Journal of Surveying Engineering, 136(2), 63-71.

Luk, F. T., & Qiao, S. (2007). ***Numerical properties of the LLL algorithm. Paper presented at the Advanced Signal Processing Algorithms***, Architectures, and Implementations XVII, Franklin T. Luk, Editors, 669703.

Luk, F. T., & Tracy, D. M. (2008). ***An improved LLL algorithm***. Linear Algebra and its Applications, 428(2-3), 441-452.

Sanzheng, Q. (2008). ***Integer least squares: sphere decoding and the LLL algorithm***. Paper presented at the Proceedings of the 2008 C3S2E conference.

Svendsen, J. (2006). ***Some properties of decorrelation techniques in the ambiguity space***. GPS Solutions, 10(1), 40-44.

Teunissen, P. J. G. (1995). ***The least - squares ambiguity decorrelation adjustment: a method for fast GPS integer ambiguity estimation***. Journal of Geodesy, 70(1), 65-82.

Teunissen, P. J. G., De Jonge, P. J., & Tiberius, C. (1995). ***The LAMBDA-Method for fast GPS Surveying***. Proceedings of International Symposium GPS technology applications, Bucharest, Romania, September 26-29, pp. 203-210.

Teunissen, P., De Jonge, P., & Tiberius, C. (1996). ***The Volume of the GPS Ambiguity Search Space and its Relevance for Integer Ambiguty Resolution.*** Proceedings of ION GPS-96, 9th International Technical Meeting of the Satellite Division of the Institute of Navigation, Kansas City, Missouri, Sept. 17-20, pp. 889-898.

Teunissen, P. J. G., De Jonge, P. J., & Tiberius, C. C. J. M. (1997). ***The least-squares ambiguity decorrelation adjustment: its performance on short GPS baselines and short observation spans.*** Journal of Geodesy, 71(10), 589-602.

Teunissen, P. J. G. (1998). ***Success probability of integer GPS ambiguity rounding and bootstrapping.*** Journal of Geodesy, 72(10), 606-612.

Verhagen, S. (2003). ***On the approximation of the integer least-squares success rate: which lower or upper bound to use.*** Journal of Global Positioning Systems, 2(2), 117-124.

Xu, P. (2001). ***Random simulation and GPS decorrelation.*** Journal of Geodesy, 75(7-8), 408-423.

Xu, P. (2002). ***Isotropic probabilistic models for directions, planes and referential systems.*** Proceedings of the Royal Society London, Series A, 458, 2017–2038.

**Biography**

Jun Wang (jun.wang@student.qut.edu.au) holds BSC from Wuhan University and MSC from Chinese Academy of Surveying and Mapping, P R China. Currently, Jun is a PhD candidate of Queensland University of Technology (QUT), Australia, researching data processing of multiple GNSS systems and signals under the financial support of China Scholarship Council (CSC) and Australian Corporative Research Center for Spatial Information (CRS-SI) scholarship programs. His research focuses on GNSS ionosphere modelling and integrity determination in real time kinematic positioning using multiple GNSS systems.